Sets and Sets and Numbers

- **Definition**: A set is any collection of objects.

- **Set Notations**
  - Sets are commonly named by capital letters.
  - Roster Form – simply list the elements in the set between curly brackets, {…}
    - Examples
      - A = { 1, 2, 3, 7, e }
    - The ellipsis (…) can be used to list an infinite set where the elements follow a pattern, i.e. B = {0, 2, 4, 6, 8, 10, …)
  - Interval Notation – when indicating a set that contains and uncountable number of elements, we can describe the set of numbers by giving the endpoints of the interval.
    - Examples
      - (0, 1) - every number between 0 and 1, but NOT INCLUDING 0 and 1
      - [0, 1] – every number between 0 and 1, and INCLUDING 0 and 1
      - Intervals may be of the form (a, b] or [a, b) as well; The union symbol may also be used to indicate more than one interval in a set.
      - $D = (0,1) \cup (4,7]$ - means D is the set of all number either between 0 and 1 (not including 0 and 1) OR the number between 4 and 7 (not including 4).
  - Set Builder Notation – describes the set using a variable and stating the mathematical properties of the elements in the set, A = {x: "properties of the elements represented by x"}
    - Examples
      - P = { z : z > 0 } means all positive numbers
      - E = { x : x ≥ 0 and x is an even integer } means the same thing as set B above under roster form.

- **Set Symbols**
  - $\in$ - "is an element of" or "is in the set"
    - If A = {1, 2, 3}, then $1 \in$ A.
  - $\notin$ - "is NOT an element of" or "is NOT in the set"
    - If A = {1, 2, 3}, then $17 \notin$ A.
  - $\cup$ - This is the symbol for Set Union. Union means "or" and is like addition.
    - If A = {1, 2, 3} and B = {3, 4, 5}, then A "or" B = A∪B = {1, 2, 3, 4, 5}
  - $\cap$ - This is the symbol for Set Intersection. Intersection means "and", and tells us to find the elements in common between two sets.
    - If A = {1, 2, 3} and B = {3, 4, 5}, then A∩B = {3} since 3 is the only element that sets A and B have in common.
  - \ - The is the symbol for set subtraction.
    - If A = {1, 2, 3} and C = {3}, then A \ B = {1, 2}.

The Hierarchy of Sets of Number

$\mathbf{N} = \{1,2,3,\ldots\}$	This set is the set of **Natural Numbers** or Counting Numbers.
$\mathbf{W} = \{0,1,2,3,\ldots\}$	This set is the set of **Whole Numbers**. Notice that $\mathbf{W} = \mathbf{N} \cup \{0\}$
$\mathbf{Z} = \{\ldots,-4,-3,-2,-1,0,1,2,3,4,\ldots\}$	This set is the set of **Integers**. Note that we can also use superscripts to indicate just the positive integers or the negative integers, $\mathbf{Z}^+$ and $\mathbf{Z}^-$. Note also that $\mathbf{Z}^+ = \mathbf{N}$.
$\mathbf{Q} = \left\{\frac{p}{q} : p \in \mathbf{Z}, q \in \mathbf{Z}, q \neq 0\right\}$	This set is the set of **Rational Numbers**, or more informally the set of fractions. Notice that the notation for **Q** uses set-builder notation. The definition says that rational numbers are fractions where the numerator can be any integer and the denominator can be any integer except 0 (since we can't divide by 0).
R = all real numbers	The set of **Real Numbers** include all of the above sets, and also the irrational numbers. **Irrational Numbers** are the numbers which are not rational, i.e. which cannot be written as a fraction. Some examples of irrational numbers are π, e, and $\sqrt{2}$.
I = irrational numbers	Note: • $\mathbf{R} = \mathbf{Q} \cup \mathbf{I}$: The real numbers equal the rational numbers union ("plus") the irrational numbers • $\mathbf{I} = \mathbf{R} \setminus \mathbf{Q}$: Irrational numbers equal the reals minus rationals • $\mathbf{Q} = \mathbf{R} \setminus \mathbf{I}$: Rational numbers equal the reals minus irrationals