Using Matlab for Numerical Evaluation of Integrals - Due Friday, April 20

Very few functions have antiderivatives which can be written down as a formula involving well-known functions. If you can't find an antiderivative, you must turn to numerical techniques. A numerical technique will provide you with a number which usually is a good approximation of the integral.

Single integrals

The matlab function `quad` allows you to perform a single integration. For example, to evaluate

$$\int_2^4 \sin(x^3 - 7x)\, dx$$

you could type:

>> syms x

```
>> f = inline(vectorize('sin(x^3 - 7*x)'),'x')
>> quad(f,2,4)
```

Double Integrals

Matlab has a command, dblquad, for integrating functions of two variables over rectangles.

For example, to evaluate

$$\int_1^2 \int_3^4 \sqrt{x^2 + y^{-2}}\; dy\; dx$$

type

>> syms x y

```
>> f = inline(vectorize('sqrt(x^2 + y^(-2))'),'x', 'y')
>> dblquad(f,1,2,3,4)
```

Note that since the variables in the inline function were specified with x first and then y, you put the x limits of integration first, followed by the y limits of integration.

Unfortunately we usually want to integrate over more general regions, not just rectangles. For some double integrals, you might be able to do the inner integration by hand. Then you have reduced the problem to a single integral which you can evaluate using quad. If you can use this method, it is fast and accurate.

For example, suppose you need to evaluate $\int\int_R y\cos(x^2)\; dA$ where R is the triangular region $\{(x, y) \mid x \geq 0, y \geq 0, x + y \leq 1\}$. It is easy to integrate this function with respect to y, so we set up the iterated integral

$$\int\int_R y\cos(x^2)\; dA = \int_0^1 \int_0^{1-x} y\cos(x^2)\; dydx$$

and evaluate the inner integral by hand as

$$(1/2)y^2\cos(x^2)|_0^{1-x} = (1-x)^2\cos(x^2)/2.$$

So

$$\int\int_R y\cos(x^2)\; dA = \int_0^1 (1-x)^2\cos(x^2)/2\, dx$$

So you could type in:

```
>> f = inline(vectorize( '(1-x)^2 *cos(x^2)/2 '),'x')
>> quad(f,0,1)
```

The second method allows you to use dblquad and integrate over a rectangle by setting your function equal to zero outside the region R. The key fact is that Matlab gives a true expression the value 1 and gives a false expression the value 0. So multiplying a function by $x + y <= 1$ is the same as setting the function equal to 0 outside the region where $x + y \leq 1$. Thus we can evaluate the above integral by using

```
>> ff = inline(vectorize('y*cos(x ^2)*(x+y<=1)' ),'x','y')
```

>> `dblquad(ff,0,1,0,1)`

Using endpoints $0 \le x \le 1, 0 \le y \le 1$ gives you the unit square containing R. Setting $f = 0$ unless $x + y \le 1$ means you just get the integral of f over the triangle R.

Note: If you are using the first method and need help with the inside integral, you can try doing the first integration symbolically using Matlab. For example, to find the antiderivative of the function $f(x, y) = y\cos(x^2 + y)$ with respect to y you can use the commands

>> `syms x y`

>> `f = y*cos(x^2+y)`

>> `int(f,y)`

On the other hand, trying to find the antiderivative of f with respect to x using

>> `int(f,x)`

gives you an antiderivative in terms of weird functions called FresnelC and Fresnel S, so would not be useful.

Triple Integrals

The command triplequad can be used for integrating functions of three variables over rectangles. If you can do the inner integration by hand to reduce the triple integral to a double integral, you can also evaluate triple integrals as above. As an example we will evaluate the integral $\int\int\int_D \sin(x^2e^y + z)\, dV$ using both methods, where D is the solid region in the first octant below the plane $3x + 2y + z = 6$.

In this case we can set the integral up as an iterated integral

$$\int_0^2 \int_0^{(6-3x)/2} \int_0^{6-3x-2y} \sin(x^2e^y + z)\, dz\, dy\, dx$$

and do the inner integration to get $\int_0^2 \int_0^{(6-3x)/2} \cos(x^2e^y) - \cos(x^2e^y + 6 - 3x - 2y)\, dy\, dx$. This is the integral over the triangular region R in the first quadrant of the xy-plane below the line $3x + 2y = 6$. You can now use the second method of the previous section to evaluate the double integral using dblquad. As your rectangle you would use $0 \le x \le 2$ and $0 \le y \le 3$. You would multiply your function by $3x + 2y <= 6$ to obtain the integral over the triangle R.

You could also evaluate this integral using triplequad as follows. Since the plane $3x + 2y + z = 6$ intersects the coordinate axes at $x = 2, y = 3, z = 6$, you will integrate over the rectangular solid region $0 \le x \le 2, 0 \le y \le 3, 0 \le z \le 6$ that contains D. You will also multiply your function by $3*x+2*y+z <= 6$ so that it is zero above the plane.

>> `ff = inline(vectorize('sin(x ^2*exp(y)+z)*(3*x+2*y+z<=6)' ),'x','y','z')`

>> `triplequad(ff,0,2,0,3,0,6)` Note - Matlab can be very slow with triplequad when you use the default settings. You can make it run faster (with less accuracy) by using

>> `triplequad(ff,0,2,0,3,0,6,.0002)`

Computer assignment - Due Friday, April 20

You must provide me with printed copies of your output with your answers clearly indicated. Also show your work, for example any inner integral calculations. As usual, you may work in groups of two or three, but no more. Warning - You need to figure out the limits of integration yourself. If you do these wrong, you will get the wrong answer.

1) Find the integral of $f(x, y) = \cos(x + e^y)$ over the triangle with vertices $(0, 0), (3, 0), (3, 1)$ in two ways. (a) Set up the double integral so that you can carry out the inner integration by hand. Then evaluate the outer integral using quad. (b) Use dblquad to evaluate the double integral directly.
2) Use either method above to find the integral of $f(x, y, z) = 1/(2 + z + xy)$ over the tetrahedron with vertices $(0, 0, 0), (1, 0, 0), (0, 2, 0), (0, 0, 1)$.