1. Find the equation $z = ax + by + c$ for the plane passing through the three points $p_1 = (0, 2, -1), p_2 = (-2, 4, 3), p_3 = (2, -1, -3)$.

2. Find the LU decomposition of $A$ with pivoting and solve the linear system $Ax = b$:

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 3 & 6 & 2 & -1 \\ 1 & 1 & -7 & 2 \\ 1 & -1 & 2 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix}.$$

3. A block matrix $D$ is called *block diagonal* if $D$ can be written as

$$D = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

with $A$ and $B$ squares matrices not necessarily of the same size, while the 0’s are zero matrices of the appropriate sizes. Prove that $D$ has an inverse if and only if $A$ and $B$ do, and

$$D^{-1} = \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix}.$$

4. True and False: (a) If $A$ is symmetric then $A^2$ is symmetric.
   (b) If $A$ is nonsingular symmetric matrix then $A^{-1}$ is symmetric.
   (c) If $A$ and $B$ are symmetric $n \times n$ matrices, so is $AB$.
   (d) If $A$ is symmetric and $D$ is diagonal then $AD$ is symmetric.

5. (a) Let $A$ be an $n \times n$ matrix. Which is faster to compute $A^2$ or $A^{-1}$.
   (b) What about $A^3$ versus $A^{-1}$?
   (c) How many operations (flops) are needed to compute $A^k$? Hint: When $k > 3$, you can get away with less than $k - 1$ matrix multiplications.
   (d) Which is faster, back substitution of multiplying $A$ by a vector?

5. Determine the rank($A$) and the decomposition $PA = LU$ for

$$A = \begin{bmatrix} 0 & 0 & 0 & 3 & 1 \\ 1 & 2 & -3 & 1 & -2 \\ 2 & 4 & -2 & 1 & -2 \end{bmatrix}.$$

6. (a) Let $V$ the space of integrable functions in $[0, 1]$. Show that the set of functions with integral zero form a subspace of $V$.
   (b) Show that the set of solutions of the ordinary differential equation $y'' + 2y' - 3y = 0$ form a subspace of the functions with two continuous derivatives. Find a basis and the dimension.
   (c) How about the nonhomogeneous ordinary differential equation $y'' + 2y' - 3y = 1$?
(d) The trace of a square matrix \(a \in \mathbb{R}^{n \times n}\) is the sum of its diagonal entries. Prove that the set of trace zero matrices is a subspace of \(\mathbb{R}^{n \times n}\).

(e) A planar vector field \(p(x, y) = (u(x, y), v(x, y))^T\) is called *irrotational* if it has zero divergence \(\text{div } p = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\). Show that the set of irrotational vector fields forms a subspace of the space of vector fields.

7. Determine whether the polynomials \(x^2 + 1, x^2 - 1, x^2 + x + 1\) span the space of quadratic polynomials \(P_2\).

8. Show that the functions \(f(x) = x\) and \(g(x) = |x|\) are linearly independent when considered as functions on all of \(\mathbb{R}\), but are linearly dependent when considered as functions defined only on \(\mathbb{R}^+ = \{x > 0\}\).

9. Given the following vectors, answer the following questions and provide a justification:

\[
\begin{align*}
v_1 &= \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \\
v_2 &= \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \\
v_3 &= \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \\
v_4 &= \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}.
\end{align*}
\]

(a) Do \((v_i)_{i=1}^4\) span \(\mathbb{R}^3\)?
(b) Are \((v_i)_{i=1}^4\) linearly independent?
(c) Do \((v_i)_{i=1}^4\) form a basis for \(\mathbb{R}^3\)? If not, is it possible to choose some subset which is a basis?
(d) What is the dimension of the span of \((v_i)_{i=1}^4\)?

10. Find basis of the four spaces \( \text{rng } A, \ker A, \text{corng } A \) and \( \text{coker } A \) of the following matrix \(A\) along with their dimensions:

\[
A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & -1 & 3 \\ 2 & 3 & 7 & 0 \end{bmatrix}.
\]

11. Show that \(v_1 = (1, 2, 0, -1)^T, v_2 = (-3, 1, 1, -1)^T, v_3 = (2, 0, -4, 3)^T\) and \(w_1 = (3, 2, -4, 2)^T, w_2 = (2, 3, -7, 4)^T, w_3 = (0, 3, -3, 1)^T\) are two bases for the same three dimensional subspace \(V\) of \(\mathbb{R}^4\).