AMSC 612 Fall 2008

NUMERICAL METHODS FOR PDE

HOMEWORK # 1 (due Tu Sep 23)

1 (25 pts). Problem 2.4 of Larsson and Thomée.

2 (25 pts). Problem 2.8 of Larsson and Thomée.

3 (20 pts). Problem 4.3 of Larsson and Thomée

4 (30 pts). Consider the two-point boundary value problem with parameter b:

$$-u'' + bu' + u = 2x \quad \text{in } (0,1), \qquad \text{with } u(0) = u(1) = 0. \tag{1}$$

(a) Find the exact solution $u(x)$ in terms of the parameter b.

(b) Write the finite difference approximation of (1) using centered differences and upwind differences on a uniform mesh with meshsize h. Write the matrix form of the system and examine how the equations would change if $u(1) = \alpha \neq 0$.

(c) Implement the finite difference method using MATLAB. To this end use the command `diag` to construct the corresponding tridiagonal matrix (type `help diag` to learn about this command). Use the backslash command \ to find the solution to `Ax=b` as `x = A\b`.

(d) Solve the linear system for $h = \frac{1}{5}2^{-k}$ for $0 \leq k \leq 5$ and $b = 0, 100$. Compute the maximum norm error at the nodes and plot it vs h in a log-log plot. Explain your findings.

(e) Plot the exact solution $u(x)$ and the computed solution as a piecewise linear function over the corresponding grid for $h = 1/20, 1/80$ and $b = 0, 100$. Draw conclusions.