AMSC 612 Fall 2008

NUMERICAL METHODS FOR PDE

HOMEWORK # 2 (due Th October 9)

1 (20 pts). Problem A.7 of Larsson and Thomée.

2 (20 pts). Problem A.11 of Larsson and Thomée

3 (20 pts). Problem 5.1 of Larsson and Thomée.

4 (20 pts). Problem 5.4 of Larsson and Thomée.

5 (20 pts). Consider $u(x) = \sqrt{x}$ over $I = [0, 1]$, and $\mathcal{T} = \{x_j\}_{j=0}^J$. The behavior $u \approx \sqrt{x}$ corresponds to a crack in a two dimensional problem (reentrant corner with internal angle $\omega = 2\pi$).

(a) Show that

$$\|u - I_{\mathcal{T}} u\|_{L^\infty(x_i, x_{i+1})} = \frac{\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right)^2}{4\left(\sqrt{x_{i+1}} + \sqrt{x_i}\right)}.$$

Conclude that $\|u - I_{\mathcal{T}} u\|_{L^\infty(I)} \geq \frac{1}{4\sqrt{N}}$ provided $\mathcal{T}$ is uniform, i.e. $h = h_i$ for all i.

(b) Suppose $\mathcal{T}$ is graded so that $x_i = (\frac{i}{N})^4$. Show that

$$\|u - I_{\mathcal{T}} u\|_{L^\infty(x_i, x_{i+1})} = \frac{1}{4N^2}\left(2 - \frac{1}{i^2 + (i-1)^2}\right).$$

Conclude that the global maximum error is $\leq \frac{1}{2N^2}$, which is the best approximation possible with $N + 1$ points. Compare with (a) and draw conclusions.