AMSC 667 (Spring 2006)

NUMERICAL ANALYSIS II

Tu-Th 11-12:15, MTH 2300 (starting 1/31/06)

INSTRUCTOR

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Office Hours: Tu 2-3, We 4-5 (or by appointment)

COURSE OUTLINE

This course deals with numerical methods for nonlinear algebraic systems, initial value problems for ordinary differential equations and boundary value problems (mostly in 1D). Each of these three parts will take 4-5 weeks.

0. Multivariable Calculus Review

- Differentiation: Frechet and Gateaux derivatives, approximations.
- Contraction Mapping theorem and applications.

1. Nonlinear Systems of Equations

- Fixed point iterations.
- Newton's method: derivation, local quadratic convergence, variations.
- Quasi-Newton methods: secant method in 1D, Broyden method, inverse Broyden method.
- Minimization: descent direction and line search, steepest descent, Newton's method, backtracking.

2. Ordinary Differential Equations

- Elements of ODE theory: linear and nonlinear ODEs, multiscales and stiff problems, stability.
- One-step methods: explicit and implicit Euler and trapezoidal method, absolute stability, truncation error, predictor-corrector formulas, error analysis.
- Explicit Runge-Kutta methods: 2nd and 4th order methods, Runge-Kutta-Fehlberg methods.
- Multistep methods: midpoint method, Gear's family, Adams family, truncation error and consistency conditions, characteristic polynomials.
- Stability and convergence: linear difference equations, root condition, convergence.
- Relative and absolute stability: midpoint method, strong root condition, regions of absolute stability.
- Multivalue methods: Nordsieck version of multistep methods.

3. Boundary Value Problems

- Model problem in 1D: Dirichlet and Neumann boundary conditions, comparison theorems.
- The finite difference method: centered second differences, approximations of Neumann condition, truncation error. M and irreducible matrices, pointwise error bounds, 2D problems, disadvantages.
- Variational formulation: bilinear forms, elements of functional analysis and Sobolev spaces, weak solutions.
- The finite element method: Galerkin procedure, discrete subspaces and basis functions, matrix formulation, existence and uniqueness, computation of stiffness and mass matrices, orthogonality and error analysis.

TEXTS

P. Deuflhard, *Newton Methods for Nonlinear Problems: Affine Invariance and Adaptive Algorithms*, Springer 2004.

C.T. Kelley, *Iterative Methods for Linear and Nonlinear Equations*, SIAM, 1995.

J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis*, Springer, 1980.

BIBLIOGRAPHY

J.E. Dennis and R.B. Schnabel, *Numerical Methods for Unsconstrained Optimization and Nonlinear Equations*, SIAM, 1996.
R.B Kellogg, *Notes of §§0 and 1 of MAPL 667.*
J.M. Ortega and W.C. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables*, Academic Press, 1970.
L.F. Shampine, *Numerical Solution of Ordinary Differential Equations*, Chapman and Hall, 1994.

EVALUATION POLICY

There will be about 6 HOMEWORKS which will amount to 40% of the final grade. The homeworks will be about 80% theoretical and 20% computational using MATLAB. There will be a penalty of 10% per day late; homeworks will not be accepted after one week. There will be a MIDTERM exam tentatively on April 6, which will amount to 30% of the grade. The FINAL exam will be comprehensive and will constitute 30% of the grade.