# Energy-Efficient Resource Allocation in Wireless Networks with Quality-of-Service Constraints

Farhad Meshkati, H. Vincent Poor, Stuart C. Schwartz, and Radu V. Balan

**Abstract**

A game-theoretic model is proposed to study the cross-layer problem of joint power and rate control with quality of service (QoS) constraints in multiple-access networks. In the proposed game, each user seeks to choose its transmit power and rate in a distributed and selfish manner in order to maximize its own utility and at the same time satisfy its QoS requirements. The user's QoS constraints are specified in terms of the average source rate and an upper bound on the average delay where the delay includes both transmission and queueing delays. . The utility function considered here measures the number of reliable bits transmitted per Joule of energy consumed and is particularly suitable for wireless networks in which energy efficiency is important. The Nash equilibrium solution for the proposed non-cooperative game is derived and a closed-form expression for the utility achieved at equilibrium is obtained. It is shown that the QoS requirements of a user translate into a "size" for the user which is an indication of the amount of network resources consumed by the user. Using this framework, the tradeoffs among throughput, delay, network capacity and energy efficiency are also studied. In addition, we give analytical expressions for users' delay profiles and quantify the delay performance of the users at Nash equilibrium.

**Index Terms**

Energy efficiency, delay, quality of service, cross-layer design, power and rate control, admission control.

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# I. Introduction

Future wireless networks are expected to support a variety of services with diverse quality of service (QoS) requirements. Because of the hostile characteristics of wireless channels and scarcity of radio resources such as power and bandwidth, efficient resource allocation schemes are necessary for design of high-performance wireless networks. The objective is to use the radio resources as efficiently as possible and at the same time satisfy the QoS requirements of the users in the network. QoS is expressed in terms of constraints on rate, delay or fidelity. Since in most practical scenarios, the users' terminals are battery-powered, energy efficient resource allocation is crucial to prolonging the battery life of the terminals.

In this work, we study the cross-layer problem of QoS-constrained joint power and rate control in wireless networks using a game-theoretic framework. We consider a multiple-access network and propose a non-cooperative game in which each user seeks to choose its transmit power and rate in such a way as to maximize its energy-efficiency (measured in bits per Joule) and at the same time satisfy its QoS requirements. The QoS constraints are in terms of the average source rate and the upper bound on the average total delay (transmission plus queueing delay). We derive the Nash equilibrium solution for the proposed game and use this framework to study trade-offs among throughput, delay, network capacity and energy efficiency. Network capacity here refers to the maximum number of users that can be accommodated by the network. While the delay QoS considered here is in terms of average delay, we also derive analytical expressions for the user's delay profile and quantify the delay performance at Nash equilibrium.

Joint power and rate control with QoS constraints have been studied extensively for multiple-access networks (see for example [1] and [2]). In [1], the authors study joint power and rate control under bit-error rate (BER) and average delay constraints. [2] considers the problem of globally optimizing the transmit power and rate to maximize throughput of non-real-time users and protect the QoS of real-time users. Neither work takes into account energy-efficiency. Recently tradeoffs between energy efficiency and delay have gained more attention. The tradeoffs in the single-user case are studied in [3]–[6]. The multiuser problem in turn is considered in [7] and [8]. In [7], the authors present a centralized scheduling scheme to transmit the arriving packets within a specific time interval such that the total energy consumed is minimized whereas in [8], a distributed ALOHA-type scheme is proposed for achieving energy-delay tradeoffs. Joint power and rate control for maximizing goodput in delay-constrained networks is studied in [9].

This work is the first study of QoS-constrained power and rate control in multiple-access networks using a game-theoretic framework. In our proposed game-theoretic model, users choose their transmit powers and rates in a *competitive* and *distributed* manner in order to maximize their energy efficiency and at the same time satisfy their delay and rate QoS requirements. Using this framework, we also analyze the the tradeoffs among throughput, delay, network capacity and energy efficiency. It should be noted that power control games have previously been studied in [10]–[17]. However, [10]–[16] do not take into account the effect of delay, and [17] only considers transmission delay and does not perform any rate control.

The remainder of this paper is organized as follows. In Section II, we describe the system model. The proposed joint power and rate control game is discussed in Section III and its Nash equilibrium solution is derived in Section IV. We then describe an admission control scheme in Section V. The users' delay performance is analyzed in Section VI. Based on our analysis, the tradeoffs among throughput, delay, network capacity and energy efficiency are studied in Section VII using numerical results. Finally, we give conclusions in Section VIII.

## II. System Model

We consider a direct-sequence code-division multiple-access (DS-CDMA) network and propose a non-cooperative (distributed) game in which each user seeks to choose its transmit power and rate to maximize its energy efficiency (measured in bits per joule) while satisfying its QoS requirements. We specify the QoS constraints of user $k$ by $(r_k, D_k)$ where $r_k$ is the average source rate and $D_k$ is the upper bound on average delay. The delay includes both queueing and transmission delays. The incoming traffic is assumed to have a Poisson distribution with parameter $\lambda_k$ which represents the average packet arrival rate with each packet consisting of $M$ bits. The source rate (in bit per second), $r_k$, is hence given by

$$r_k = M\lambda_k. \tag{1}$$

The user transmits the arriving packets at a rate $R_k$ (bps) and with a transmit power equal to $p_k$ Watts. We consider an automatic-repeat-request (ARQ) mechanism in which the user keeps retransmitting a packet until the packet is received at the access point without any errors. The incoming packets are assumed to be stored in a queue and transmitted in a first-in-first-out (FIFO) fashion. The packet transmission time for user $k$ is defined as

$$\tau_k = \frac{M}{R_k} + \epsilon_k \simeq \frac{M}{R_k}, \tag{2}$$

where $\epsilon_k$ represents the time taken for the user to receive an ACK/NACK from the access point. We assume $\epsilon_k$ is negligible compared to $\frac{M}{R_k}$. The packet success probability (per transmission) is represented by $f(\gamma_k)$ where $\gamma_k$ is the received signal-to-interference-plus-noise ratio (SINR) for user $k$. The retransmissions are assumed to be independent. The packet success rate, $f(\gamma)$, is assumed to be increasing and S-shaped (sigmoidal) with $f(0) = 0$ and $f(\infty) = 1$. This is a valid assumption for many practical scenarios as long as the packet size is reasonably large (e.g., $M = 100$ bits).

We can represent the combination of user $k$'s queue and wireless link as an M/G/1 queue, as shown in Fig. 1 where the traffic is Poisson with parameter $\lambda_k$ (in packets per second) and the service time, $S_k$, has the following probability mass function (PMF):

$$\Pr\{S_k = m\tau_k\} = f(\gamma_k)\,(1 - f(\gamma_k))^{m-1} \quad \text{for } m = 1, 2, \cdots \tag{3}$$

As a result, we have

$$\mathbb{E}\{S_k\} = \sum_{m=1}^{\infty} m\tau_k\,(1 - f(\gamma_k))^{m-1} = \frac{\tau_k}{f(\gamma_k)}. \tag{4}$$

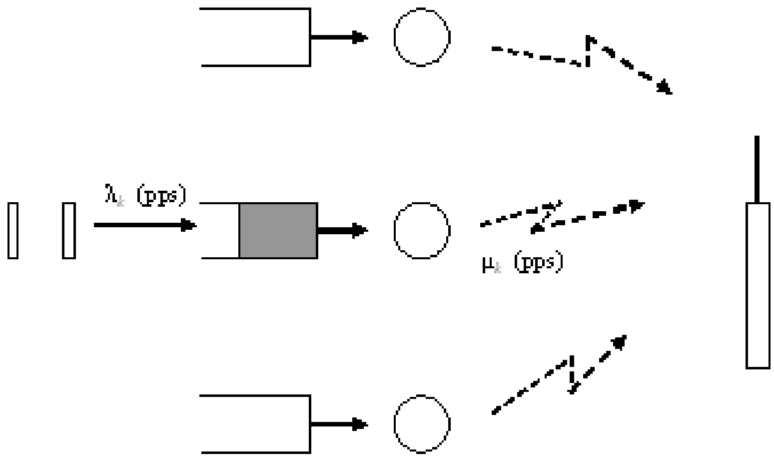


Fig. 1. System model based on an M/G/1 queue.

Consequently, the service rate, $\mu_k$, is given by

$$\mu_k = \frac{1}{\mathbb{E}\{S_k\}} = \frac{f(\gamma_k)}{\tau_k}, \quad (5)$$

and the load factor $\rho_k = \frac{\lambda_k}{\mu_k} = \frac{\lambda_k \tau_k}{f(\gamma_k)}$.

To keep the queue of user $k$ stable, we must have $\rho_k < 1$ or $f(\gamma_k) > \lambda_k \tau_k$. Now, let $W_k$ be a random variable representing the total packet delay for user $k$. This delay includes the time the packet spends in the queue, $W_k^{(q)}$, as well as the service time, $S_k$. Hence, we have

$$W_k = W_k^{(q)} + S_k. \quad (6)$$

It is known that for an M/G/1 queue the average wait time (including the queueing and service time) is given by

$$\bar{W}_k = \frac{\bar{L}_k}{\lambda_k}, \quad (7)$$

where $\bar{L}_k = \rho_k + \frac{\rho_k^2 + \lambda_k^2 \sigma_{S_k}^2}{2(1-\rho_k)}$ with $\sigma_{S_k}^2$ being the variance of the service time [18]. Therefore, the average packet delay for user $k$ is given by

$$\bar{W}_k = \tau_k \left( \frac{1 - \frac{\lambda_k \tau_k}{2}}{f(\gamma_k) - \lambda_k \tau_k} \right) \quad \text{with } f(\gamma_k) > \lambda_k \tau_k. \quad (8)$$

We require the average delay for user $k$'s packets to be less than or equal to $D_k$. This translates to

$$\bar{W}_k \leq D_k \quad (9)$$

or

$$f(\gamma_k) \geq \lambda_k \tau_k + \frac{\tau_k}{D_k} - \frac{\lambda_k \tau_k^2}{2 D_k}. \quad (10)$$

However, since $f(\gamma_k) < 1$, (10) is possible only if[1]

$$\lambda_k \tau_k < \frac{\frac{D_k}{\tau_k} - 1}{\frac{D_k}{\tau_k} - \frac{1}{2}}. \quad (11)$$

[1]Note that $f(\gamma) = 1$ requires an infinite SINR which is not practical.

This means that $r_k = M\lambda_k$ and $D_k$ are feasible only if they satisfy (11). Note that the upper bound on the average delay (i.e., $D_k$) cannot be smaller than transmission time $\tau_k$, that is, $\frac{D_k}{\tau_k} > 1$. This automatically implies that $\lambda_k \tau_k + \frac{\tau_k}{D_k} - \frac{\lambda_k \tau_k^2}{2D_k} > 0$.

Let us define $\eta_k = \lambda_k \tau_k + \frac{\tau_k}{D_k} - \frac{\lambda_k \tau_k^2}{2D_k}$. Then, (10) is equivalent to the condition $\gamma \geq \hat{\gamma}_k$ where

$$\hat{\gamma}_k = f^{-1}(\eta_k). \tag{12}$$

This means that the delay constraint in (9) translated into a lower bound on the output SINR.

## III. The Joint Power and Rate Control Game

Consider the non-cooperative joint power and rate control game (PRCG) $G = [\mathcal{K}, \{A_k\}, \{u_k\}]$ where $\mathcal{K} = \{1, 2, \cdots, K\}$ is the set of users, $A_k = [0, P_{max}] \times [0, B]$ is the strategy set for user $k$ with a strategy corresponding to a choice of transmit power and transmit rate, and $u_k$ is the utility function for user $k$. Here, $P_{max}$ and $B$ are the maximum transmit power and the system bandwidth, respectively. For the sake simplicity, throughout this paper, we assume $P_{max}$ is large. Each user chooses its transmit power and rate in order to maximize its own utility while satisfying its QoS requirements. The utility function for a user is defined as the ratio of the user's goodput to its transmit power, i.e.,

$$u_k = \frac{T_k}{p_k}, \tag{13}$$

where the goodput $T_k$ is the number of bits that is transmitted successfully per second and is given by

$$T_k = R_k f(\gamma_k). \tag{14}$$

Therefore, the utility function for user $k$ is given by

$$u_k = R_k \frac{f(\gamma_k)}{p_k}. \tag{15}$$

This utility function has units of bits per Joule and is particularly suitable for wireless networks where energy efficiency is important.

Fixing other users' transmit powers and rates, the utility-maximizing strategy for user $k$ is given by the solution of the following constrained maximization:

$$\max_{p_k, R_k} u_k \quad \text{s.t.} \quad \bar{W}_k \leq D_k \ , \tag{16}$$

or equivalently

$$\max_{p_k, R_k} u_k \quad \text{s.t.} \quad \gamma_k > \hat{\gamma}_k \quad \text{and} \quad \frac{r_k}{R_k} < \frac{\frac{D_k R_k}{M} - 1}{\frac{D_k R_k}{M} - \frac{1}{2}} \tag{17}$$

where $\hat{\gamma}_k = f^{-1}(\eta_k)$ and

$$\eta_k = \frac{r_k}{R_k} + \frac{M}{D_k R_k} - \frac{M r_k}{2 D_k R_k^2} \ . \tag{18}$$

Note that for a matched filter receiver and with random spreading sequences, the received SINR is approximately given by

$$\gamma_k = \left(\frac{B}{R_k}\right)\frac{p_k h_k}{\sigma^2 + \sum_{j\neq k} p_j h_j}, \tag{19}$$

where $h_k$ is the channel gain for user $k$ and $\sigma^2$ is the noise power in the bandwidth $B$.

Let us first look at the maximization in (17) without any constraints, i.e.,

$$\max_{p_k, R_k} u_k \quad \equiv \max_{p_k, R_k} R_k \frac{f(\gamma_k)}{p_k}. \tag{20}$$

*Proposition 1:* The unconstrained utility maximization in (20) has an infinite number of solutions. More specifically, any combination of $p_k$ and $R_k$ that achieves an output SINR equal to $\gamma^*$, the solution to $f(\gamma) = \gamma f'(\gamma)$, maximizes $u_k$.

*Proof:* Let $\tilde{p}_k$ and $\tilde{R}_k$ be any power-rate combination such that

$$\left(\frac{B}{\tilde{R}_k}\right)\frac{\tilde{p}_k h_k}{\sigma^2 + \sum_{j\neq k} p_j h_j} = \tilde{\gamma}$$

or

$$\frac{\tilde{R}_k}{\tilde{p}_k} = \frac{B\hat{h}_k}{\tilde{\gamma}} \tag{21}$$

where

$$\hat{h}_k = \frac{h_k}{\sigma^2 + \sum_{j\neq k} p_j h_j} \ . \tag{22}$$

Consequently, we have

$$\tilde{u}_k = \tilde{R}_k \frac{f(\tilde{\gamma})}{\tilde{p}_k} = \frac{B\hat{h}_k}{\tilde{\gamma}} f(\tilde{\gamma}). \tag{23}$$

This means that when other users' powers and rates are fixed (i.e., fixed $\hat{h}_k$), user $k$'s utility depends only on $\tilde{\gamma}$ and is independent of the specific values of $p_k$ and $R_k$. In addition, by taking the derivative of $\frac{f(\gamma)}{\gamma}$ with respect to $\gamma$ and equating it to zero, it can be shown that $\frac{f(\gamma)}{\gamma}$ is maximized when $\gamma = \gamma^*$, the (unique) positive solution of $f(\gamma) = \gamma f'(\gamma)$. Therefore, $u_k$ is maximized for any combination of $p_k$ and $R_k$ for which $\gamma_k = \gamma^*$. This means that there are infinite number of solutions for the unconstrained maximization in (20). ∎

Now, in order to obtain a closed-form solution for the maximization in (17), let us assume that

$$f(\gamma_k) = (1 - e^{-\gamma_k})^M. \tag{24}$$

This serves as an approximation for the packet success rate that is very reasonable for moderate to large values of $M$. Given the function in (24), we have

$$\hat{\gamma}_k = -\ln(1 - \eta_k^{\frac{1}{M}}) \tag{25}$$

where $\eta_k$ is given by (18).

The second constraint in (17) can equivalently be expressed as

$$R_k > \left(\frac{M}{D_k}\right)\frac{1 + D_k\lambda_k + \sqrt{1 + D_k^2\lambda_k^2}}{2}. \tag{26}$$

Therefore, the maximization in (17) is equivalent to

$$\begin{aligned} \max_{p_k,R_k} \quad & R_k \frac{f(\gamma_k)}{p_k} \\ \text{s.t.} \quad \gamma_k \quad > \quad & -\ln(1-\eta_k^{\frac{1}{M}}) \\ \text{and} \quad R_k \quad > \quad & \left(\frac{M}{D_k}\right) \frac{1+D_k\lambda_k+\sqrt{1+D_k^2\lambda_k^2}}{2} \ . \end{aligned} \tag{27}$$

Let us define

$$\Omega_k^{\infty} = \left(\frac{M}{D_k}\right) \frac{1+D_k\lambda_k+\sqrt{1+D_k^2\lambda_k^2}}{2}.$$

Note that for $R_k = \Omega_k^{\infty}$, we have $\eta_k = 1$ and hence $\hat{\gamma}_k = \infty$. Also, define $\Omega_k^*$ as the rate for which $\hat{\gamma}_k = \gamma^*$, i.e.,

$$\Omega_k^* = \left(\frac{M}{D_k}\right) \frac{1+D_k\lambda_k+\sqrt{1+D_k^2\lambda_k^2+2(1-f^*)D_k\lambda_k}}{2f^*} \tag{28}$$

where $f^* = f(\gamma^*)$. Since for all practical choices of $M$, $D_k$ and $\lambda_k$, we have $\Omega_k^{\infty} > 1$, then $\hat{\gamma}_k$ is a decreasing function of $R_k$ for all $R_k > \Omega_k^{\infty}$. Therefore, $\hat{\gamma}_k > \gamma^*$ for all $\Omega_k^{\infty} < R_k < \Omega_k^*$. This means that based on (23), user $k$ has no incentive to transmit at a rate smaller than $\Omega_k^*$. Furthermore, based on Proposition 1, any combination of $p_k$ and $R_k \geq \Omega_k^*$ that results in an output SINR equal to $\gamma^*$ is a solution to the constrained maximization in (17). Note that when $R_k = \Omega_k^*$ and $\gamma_k = \gamma^*$, we have $\bar{W}_k = D_k$.

## IV. Nash Equilibrium for the PRCG

For a non-cooperative game, a Nash equilibrium is defined as a set of strategies for which no user can unilaterally improve its own utility [19]. We saw in Section III that for our proposed non-cooperative game, each user has infinitely many strategies that maximize the user's utility. In particular, any combination of $p_k$ and $R_k$ for which $\gamma_k = \gamma^*$ and $R_k \geq \Omega_k^*$ is a best-response strategy.

*Proposition 2:* If $\sum_{k=1}^{K} \frac{1}{1+\frac{B}{\Omega_k^*\gamma^*}} < 1$, then the PRCG has at least one Nash equilibrium. Furthermore, when there are more than one Nash equilibrium, the most *efficient* one corresponds to $(p_k^*, R_k^*)$ where $R_k^* = \Omega_k^*$ and $p_k^* = \frac{\sigma^2\gamma^*}{h_k}\left(\frac{1-\frac{1}{1+\frac{B}{\Omega_k^*\gamma^*}}}{1-\sum_{j=1}^{K}\frac{1}{1+\frac{B}{\Omega_j^*\gamma^*}}}\right)$ for $k = 1, \cdots, K$.

*Proof:* If $\sum_{j=1}^{K} \frac{1}{1+\frac{B}{\Omega_j^*\gamma^*}} < 1$ then $p_k^* = \frac{\sigma^2\gamma^*}{h_k}\left(\frac{1-\frac{1}{1+\frac{B}{\Omega_k^*\gamma^*}}}{1-\sum_{j=1}^{K}\frac{1}{1+\frac{B}{\Omega_j^*\gamma^*}}}\right)$ is positive and finite. Now, if we let $p_k = p_k^*$ and $R_k = \Omega_k^*$, then the output SINR for all the users will be equal to $\gamma^*$ which means every user is using its best-response strategy. Therefore, $(p_k^*, R_k^*)$ for $k = 1, \cdots, K$ is a Nash equilibrium.

More generally, if we let $R_k = \tilde{R}_k \geq \Omega_k^*$ and provided that $\sum_{j=1}^{K} \frac{1}{1+\frac{B}{\tilde{R}_j\gamma^*}} < 1$, then $(\tilde{p}_k, \tilde{R}_k)$ is a Nash equilibrium where $\tilde{p}_k = \frac{\sigma^2\gamma^*}{h_k}\left(\frac{1-\frac{1}{1+\frac{B}{\tilde{R}_k\gamma^*}}}{1-\sum_{j=1}^{K}\frac{1}{1+\frac{B}{\tilde{R}_j\gamma^*}}}\right)$.

Based on (15), at Nash equilibrium, the utility of user $k$ is given by

$$\begin{aligned} u_k &= \frac{Bf(\gamma^*)h_k}{\sigma^2\gamma^*}\left(\frac{1-\sum_{j=1}^{K}\frac{1}{1+\frac{B}{\tilde{R}_j\gamma^*}}}{1-\frac{1}{1+\frac{B}{\tilde{R}_k\gamma^*}}}\right) \\ &= \frac{Bf(\gamma^*)h_k}{\sigma^2\gamma^*}\left(1-\frac{\sum_{j\neq k}\frac{1}{1+\frac{B}{\tilde{R}_j\gamma^*}}}{1-\frac{1}{1+\frac{B}{\tilde{R}_k\gamma^*}}}\right) . \end{aligned} \tag{29}$$

Therefore, the Nash equilibrium with the smallest $\tilde{R}_k$ achieves the largest utility. A higher transmission rate for a user requires a larger transmit power by that user to achieve $\gamma^*$. This not only reduces the user's utility but also causes more interference for other users in the network and forces them to raise their transmit powers as well which will result in a reduction in their utilities. This means that the Nash equilibrium with $R_k = \Omega_k^*$ and $p_k^*$ for $k = 1, \cdots, K$ is the most efficient Nash equilibrium. ∎

Based on the feasibility condition given by Proposition 2, i.e.,

$$\sum_{k=1}^{K}\frac{1}{1+\frac{B}{\Omega_k^*\gamma^*}} < 1, \tag{30}$$

let us define the "size" of user $k$ as

$$\Phi_k^* = \frac{1}{1+\frac{B}{\Omega_k^*\gamma^*}} . \tag{31}$$

Therefore, the feasibility condition in (30) can be written as

$$\sum_{k=1}^{K}\Phi_k^* < 1. \tag{32}$$

Note that the QoS requirements of user $k$ (i.e., its source rate $r_k$ and delay constraint $D_k$) uniquely determine $\Omega_k^*$ through (28) and, in turn, determine the size of the user (i.e., $\Phi_k^*$) through (31). The size of a user is basically an indication of the amount of network resources consumed by that user. A larger source rate or a tighter delay constraint for a user increases the size of the user. The network can accommodate a set of users if and only if their total size is less than 1. In Section VII, we use this framework to study the tradeoffs among throughput, delay, network capacity and energy efficiency.

## V. Admission Control

In Section IV, we defined the "size" of a user based on its QoS requirements. Before joining the network, each user calculates its size using (31) and announces it to the access point. According to (32), the access point admits those users whose total size is less than 1. While the goal of each user is to maximize its own energy efficiency, a more sophisticated admission control can be performed to maximize the total network utility. In other words, out of the $K$ users, the access point can choose those users for which the total network utility is the largest, i.e.,

$$\max_{\mathcal{L}\subset\{1,\cdots,K\}}\sum_{\ell\in\mathcal{L}}u_\ell \tag{33}$$

under the constraint that $\sum_{\ell\in\mathcal{L}}\Phi_\ell^* < 1$.

Based on (29), the utility of user $\ell$ at the efficient Nash equilibrium is given by

$$u_\ell = \left(\frac{Bh_\ell f(\gamma^*)}{\sigma^2\gamma^*}\right)\frac{1-\sum_{i\in\mathcal{L}}\Phi_i^*}{1-\Phi_\ell^*} \ . \tag{34}$$

As a result, (33) becomes

$$\max_{\mathcal{L}\subset\{1,\cdots,K\}} \sum_{\ell\in\mathcal{L}} h_\ell \frac{1-\sum_{i\in\mathcal{L}}\Phi_i^*}{1-\Phi_\ell^*}$$

or equivalently

$$\max_{\mathcal{L}\subset\{1,\cdots,K\}} \left(1-\sum_{i\in\mathcal{L}}\Phi_i^*\right)\sum_{\ell\in\mathcal{L}}\frac{h_\ell}{1-\Phi_\ell^*} \tag{35}$$

under the constraint that $\sum_{\ell\in\mathcal{L}}\Phi_\ell^* < 1$.

In general, obtaining a closed-form solution for (35) is difficult. Instead, in order to gain some insight, let us consider the special case in which all users are the same distance from the access point. We first consider the scenario in which the users have identical QoS requirements (i.e., $\Phi_1^* = \cdots = \Phi_K^* = \Phi^*$). If we replace $\sum_{\ell=1}^{L} h_\ell$ by $L\mathbb{E}\{h\}$, then (35) becomes

$$\max_{L} \frac{\mathbb{E}\{h\}(L-L^2\Phi^*)}{1-\Phi^*}. \tag{36}$$

Therefore, the optimal number of users for maximizing the total utility in the network is $L = \left[\frac{1}{2\Phi^*}\right]$ where $[x]$ represents the integer nearest to $x$.

Now consider another scenario in which there are $C$ classes of users. The users in class $c$ are assumed to all have the same QoS requirements and hence the same size, $\Phi^{*(c)}$. Since we are assuming that all the users have the same distance from the access point, they all have the same channel gains. Now, if the access point admits $L^{(c)}$ users from class $c$ then the total utility is given by

$$u_T = \left(\frac{Bhf(\gamma^*)}{\sigma^2\gamma^*}\right)\left(1-\sum_{c=1}^{C}L^{(c)}\Phi^{*(c)}\right)\left(\sum_{c=1}^{C}\frac{L^{(c)}}{1-\Phi^{*(c)}}\right)$$

provided that $\sum_{c=1}^{C}L^{(c)}\Phi^{*(c)} < 1$. Without loss of generality, let us assume that $\Phi^{*(1)} < \Phi^{*(2)} < \cdots < \Phi^{*(C)}$. It can be shown that $u_T$ is maximized when $L^{(1)} = \left[\frac{1}{2\Phi^{*(1)}}\right]$ with $L^{(c)} = 0$ for $c = 2, 3, \cdots, C$. This is because adding a user from class 1 is always more beneficial in terms of increasing the total utility than adding a user from any other class. Therefore, in order to maximize the total utility in the network, the access point should admit only users from the class with the smallest size. Of course, this may not be practical. In the next section, we demonstrate the loss in network energy efficiency if a suboptimal admission control strategy is used.

## VI. Delay Performance

In Section II, we defined the delay requirement of a user as an upper bound on the average total packet delay for that user where the total delay, $W_k$, is given by the sum of the queueing delay and service time. We have considered a scenario in which users choose their transmit powers and rates in a selfish and distributed manner such that they maximize their own energy efficiency while satisfying their delay requirements. In Section IV, we showed that at the *efficient* Nash equilibrium, the transmit power and rate of a user are such that the delay bound is met with equality. However, it would be useful to obtain the delay profile of a user so that the deviations of the true delay from the average value can be quantified. More specifically, we would like to find a closed-form expression for $\Pr\{W_k \leq c\}$ for all $c$.

To that end, let us define $w_k(t)$ as the probability density function (PDF) of $W_k$. Then, we have

$$\Pr\{W_k \leq c\} = \int_0^c w_k(t)\mathrm{d}t \ . \tag{37}$$

Let $W_k^*(s)$ represent the Laplace transform for $w_k(t)$, i.e.,

$$W_k^*(s) = \int_0^\infty e^{-st} w_k(t)\mathrm{d}t \ . \tag{38}$$

It is known that for M/G/1 queues, we have

$$W_k^*(s) = \frac{(1-\rho_k)sB_k^*(s)}{s-\lambda_k[1-B_k^*(s)]} \tag{39}$$

where $B_k^*(s) = \int_0^\infty e^{-st} b_k(t)\mathrm{d}t$ with $b_k(t)$ being the PDF of the service time $S_k$ [18]. Based on (3), $b_k(t)$ is given by

$$b_k(t) = \sum_{m=1}^{\infty} f(\gamma_k)\left(1-f(\gamma_k)\right)^{m-1} \delta(t-m\tau_k) \tag{40}$$

where $\delta(\cdot)$ is the Dirac delta function. Therefore, we have

$$B_k^*(s) = \frac{f(\gamma_k)}{e^{s\tau_k}-1+f(\gamma_k)}. \tag{41}$$

As a result,

$$W_k^*(s) = \frac{(1-\rho_k)f(\gamma_k)s}{s\left(e^{s\tau_k}-1+f(\gamma_k)\right)-\lambda_k\left(e^{s\tau_k}-1\right)}. \tag{42}$$

However, obtaining a closed-form expression for $w_k(t)$ based on $W_k^*(s)$ in (42) is very difficult. But, recall from Section II that

$$W_k = W_k^{(q)} + S_k.$$

Based on this we have

$$W_k^{(q)*}(s) = \frac{W_k^*(s)}{B_k^*(s)} = \frac{(1-\rho_k)s\left(e^{s\tau_k}-1+f(\gamma_k)\right)}{s\left(e^{s\tau_k}-1+f(\gamma_k)\right)-\lambda_k\left(e^{s\tau_k}-1\right)}. \tag{43}$$

While finding the inverse Laplace transform of (43) is als difficult, we will shortly derive an accurate approximation for $w_k^{(q)}(t)$. Before doing that, let us first obtain the mean and variance of $W_k^{(q)}$ and $S_k$. For simplicity of notation, we will drop the subscript $k$ but it should be noted that all of our results are user dependent. Also, we replace $f(\gamma)$ by $f$.

Based on (3), the mean and variance of $S$ are, respectively, given by

$$\bar{S} = \frac{\tau}{f} \tag{44}$$

and

$$\sigma_S^2 = \frac{\tau^2}{f^2}(1-f). \tag{45}$$

From the known properties of M/G/1 queues [18], the mean and variance of $W^{(q)}$ are, respectively, given by

$$\bar{W}^{(q)} = \frac{\tau}{f}\left[\frac{(1-\frac{f}{2})\left(\frac{\lambda\tau}{f}\right)}{1-\frac{\lambda\tau}{f}}\right] \tag{46}$$

and

$$\sigma^2_{W^{(q)}} = \mathbb{E}\left\{W^{(q)^2}\right\} - \bar{W}^{(q)^2} = \frac{\lambda}{1-\rho}\left[\bar{W}^{(q)}\mathbb{E}\{S^2\} + \frac{\mathbb{E}\{S^3\}}{3}\right] - \bar{W}^{(q)^2}.$$

After some manipulations, it can be shown that the variance of $W^{(q)}$ is given by

$$\sigma^2_{W^{(q)}} = \frac{\tau^2}{f^2}(1-f)\left[\frac{1}{\left(1-\frac{\lambda\tau}{f}\right)^2} + \frac{f^2\left(\frac{\lambda\tau}{f}\right)\left(4-\frac{\lambda\tau}{f}\right)}{12(1-f)(1-\frac{\lambda\tau}{f})^2} - 1\right]. \tag{47}$$

To gain some insights into the contributions of the queueing delay and service time to the overall delay, let us define

$$\nu = \frac{\bar{W}^{(q)}}{\bar{S}}$$

and

$$\chi = \sqrt{\frac{\sigma^2_{W^{(q)}}}{\sigma^2_S}}.$$

Then, we have

$$\nu = \frac{(1-\frac{f}{2})\left(\frac{\lambda\tau}{f}\right)}{1-\frac{\lambda\tau}{f}}\ , \tag{48}$$

and

$$\chi = \left[\frac{1}{\left(1-\frac{\lambda\tau}{f}\right)^2} + \frac{f^2\left(\frac{\lambda\tau}{f}\right)\left(4-\frac{\lambda\tau}{f}\right)}{12(1-f)(1-\frac{\lambda\tau}{f})^2} - 1\right]^{1/2} \tag{49}$$

At the efficient Nash equilibrium, we have $\tau = \frac{M}{\Omega^*}$ and $\gamma = \gamma^*$. Therefore, based on (28), we have

$$\frac{\lambda\tau}{f} = 2\left[1 + \frac{1}{D\lambda} + \sqrt{1 + \frac{2(1-f^*)}{D\lambda} + \left(\frac{1}{D\lambda}\right)^2}\right]^{-1}. \tag{50}$$

Since $f^*$ is fixed and (50) only depends on the product $D\lambda$, then $\nu$ and $\chi$ also depend only on the product of $D$ and $\lambda$, not their individual values. Recall that $\lambda$ is the average source rate (in packets per second) and $D$ is the average delay bound. Together, they specify the QoS requirements of a user. Let $d = D\lambda$. So, for example, if the packet size $M$ is 100 bits, a source rate of $r = 50$kbps results in $\lambda = 500$pps. Then if the delay bound $D$ is 50ms, we have $d = 25$. Fig. 2 shows the plots of $\nu$ and $\chi$ versus $d$.

Two important observations can be made from Fig. 2. First of all, for moderate and large values of $d$ (e.g., $d > 10$), the average delay is dominated by the average wait time in the queue (i.e., $\bar{W}^{(q)}$). When $d$ is small, the average wait time in the queue and the average service time are comparable. For very small values of $d$, the service time dominates the total delay. Secondly, for most values of $d$ (i.e., $d > 4$), the standard deviation of $W^{(q)}$ is at least ten times larger than that of $S$. This means that the variations in the total delay are caused mainly by the variations in $\bar{W}^{(q)}$. Therefore, in many cases, the variations in the total delay can be accurately approximated by the variations in the queuing delay.

Now let $w^{(q)}(t)$ be the PDF of the queueing delay. According to (43), the Laplace transform of $w^{(q)}(t)$ is given by

$$W^{(q)*}(s) = \frac{(1-\rho)s\left(e^{s\tau}-1+f\right)}{s(e^{s\tau}-1+f)-\lambda(e^{s\tau}-1)}.$$

We can equivalently write $W^{(q)*}(s)$ as

$$W^{(q)*}(s) = P_0(s) + P_1(s) + P_2(s)$$

where

$$P_0(s) = (1-\rho), \tag{51}$$

$$P_1(s) = \frac{(1-\rho)\lambda(e^{s\tau}-1)}{s(e^{s\tau}-1+f)} \tag{52}$$

and

$$P_2(s) = \frac{(1-\rho)\lambda^2(e^{s\tau}-1)^2}{s\left[s(e^{s\tau}-1+f)-\lambda(e^{s\tau}-1)\right](e^{s\tau}-1+f)}. \tag{53}$$

Based on (51), we have

$$p_1(t) = (1-\rho)\delta(t). \tag{54}$$

*Proposition 3:* The inverse Laplace transform of (52) is given by

$$p_1(t) = \lambda(1-\rho)(1-f)^{\lfloor \frac{t}{\tau} \rfloor}, \tag{55}$$

where $\lfloor x \rfloor$ represents the nearest integer smaller than $x$.

*Proof:* See the appendix for the proof. ■

As a result of Proposition 3, we have

$$w^{(q)}(t) = (1-\rho)\delta(t) + \lambda(1-\rho)(1-f)^{\lfloor \frac{t}{\tau} \rfloor} + p_2(t). \tag{56}$$

Now if we restrict our attention to $0 \le t \le t_{max}$ where $t_{max} >> D$, then we can approximate $p_2(t)$ numerically using the following:

$$P_2(i\omega) = \int_0^{t_{max}} p_2(t)e^{-i\omega t}\mathrm{d}t \simeq \sum_{n=0}^{N-1} p_2\left(\frac{t_{max}}{N}n\right)e^{-i\omega\frac{t_{max}}{N}n}\left(\frac{t_{max}}{N}\right)$$

or

$$\left(\frac{N}{t_{max}}\right)P_2(i\omega) = \sum_{n=0}^{N-1} p_2\left(\frac{t_{max}}{N}n\right)e^{-i\omega\frac{t_{max}}{N}n}.$$

Now, since the FFT of a discrete signal $z_n$ is given by

$$Z_k = \sum_{n=0}^{N-1} z_n e^{-i\frac{2\pi kn}{N}},$$

$p_2\left(\frac{t_{max}}{N}n\right)$ can be obtained by taking the IFFT of $\left(\frac{N}{t_{max}}\right)P_2(s)|_{s=i\frac{2\pi k}{100D}}$ [2]. In Section VII, we use this approximation along with (56) to obtain $w^{(q)}(t)$ and, consequently, approximate $\Pr\{W^{(q)} \leq c\}$. This allows us to quantify the delay performance of the users at Nash equilibrium.

## VII. Numerical Results

Let us consider the uplink of a DS-CDMA system with a total bandwidth of 5MHz (i.e. $B = 5$MHz). Each user in the network has a set of QoS requirements expressed as $(r_k, D_k)$ where $r_k$ is the source rate and $D_k$ is the delay requirement (upper bound on the average total delay) for user $k$. As explained in Section IV, the QoS parameters of a user define a "size" for that user, denoted by $\Phi_k^*$ given by (31). Before a user starts transmitting, it must announce its size to the access point. Based on the particular admission policy, the access point decides whether or not to admit the user. Throughout this section, we assume that the admitted users choose the transmit powers and rates that correspond to their efficient Nash equilibrium.

Fig. 3 shows the size of a user as a function of the user's source rate and for different delay requirements. It is seen that the higher the source rate and the tighter the delay requirement, the larger the size. For example, a user with a source rate of 50kbps and a maximum average delay of 50ms (i.e., $r = 50$kbps and $D = 50$ms) has a size equal to 0.072. Fig. 4 shows the transmission rate at equilibrium as a function of the source rate for various delay requirements. It is seen that for $r = 50$kbps and $D = 50$ms, the transmission rate is equal to 59.65kbps.

Now, let us assume that all users in the network have the same QoS requirements, which means that all the users have the same size. Based on (32), we can calculate the maximum number of users whose QoS requirements can be accommodated (i.e., network capacity). Fig. 5 shows the network capacity as a function of the source rate for different delay requirements. For example, it can be seen from the figure that the network can accommodate at most 13 users if the users have a source rate of 50kbps and a delay constraint of 50ms. As the source rate increases and the delay bound becomes tighter, the number of users that can be accommodate by the network reduces. Eventually, as the source rate becomes very large, only one user can be accommodated by the network.

We can also plot the total throughput and the total goodput (i.e., reliable throughput) of the network. Figs. 6 and 7, respectively, show the total throughput and the total goodput as a function of the source rate for different delay requirements. It can be seen that when the source rate is 50kbps and the delay constraint is 50ms, the total throughput is 775.5kbps and the total goodput is 650kbps. When we reach the point in which only one user can be admitted in the network (not shown in the figures), then both the throughput and goodput increase linearly with the source rate. In this case, the goodput becomes equal to the source rate.

Now to study admission control, let us consider a network with three different classes of users/sources:

1) Class $A$ users for which the source rate is low and the delay is tight. For this family, we set $r^{(A)} = 5$kbps and $D^{(A)} = 10$ms.

[2]Since $p_2(t)$ is real, before taking the IFFT, we have to make sure that the samples of $P_2(s)$ satisfy the symmetry properties associated with the FFT of real signals.

TABLE I

PERCENTAGE LOSS IN THE TOTAL NETWORK UTILITY FOR DIFFERENT CHOICES OF $L^{(A)}, L^{(B)}$ AND $L^{(C)}$.

| $L^{(A)}$ | $L^{(B)}$ | $L^{(C)}$ | Loss in total utility |
|---|---|---|---|
| 25 | 0 | 0 | 0 |
| 23 | 1 | 0 | 10% |
| 20 | 0 | 1 | 30% |
| 18 | 1 | 1 | 38% |
| 0 | 7 | 0 | 71% |
| 0 | 0 | 3 | 87% |

2) Class $B$ users for which the source rate is high and the delay is loose. For this family, we set $r^{(B)} = 50$kbps and $D^{(B)} = 50$ms.
3) Class $C$ users for which the source rate is very high and the delay is very loose. For this family, we set $r^{(C)} = 150$kbps and $D^{(C)} = 1000$ms.

We can calculate the size of a user in each class using (31) to get $\Phi^{*^{(A)}} = 0.0198$, $\Phi^{*^{(B)}} = 0.0718$, and $\Phi^{*^{(C)}} = 0.1848$. This means that users in classes $B$ and $C$ respectively consume approximately 3.6 and 9.3 times as much resources as a user in class $A$.

For the purpose of illustration and to keep the comparison fair, let us assume that there are a large number of users in each class and that they all are the same distance from the access point (i.e., they all have the same channel gain). The access point receives requests from the users and has to decide which ones to admit in order to maximize the total utility in the network (see (35)). We know from Section V that since users in class $A$ have the smallest size, the total utility is maximized if the access point picks users from class $A$ only with $L^{(A)} = \left\lfloor 1/2\Phi^{*(A)} \right\rfloor = 25$. However, this may not be a useful solution. We may be more interested in cases where more than one class of users are admitted. Table I shows the percentage loss in the total utility (energy efficiency) for several choices of $L^{(A)}, L^{(B)}$ and $L^{(C)}$. It is observed that admitting "large" users into the network results in significant reductions in the energy efficiency and capacity of the network.

Let us now focus on the delay profile of a user in class B. For this user, we have $r^{(B)} = 50$kbps (or $\lambda^{(B)} = 500$pps) and $D^{(B)} = 50$ms. Therefore, $d^{(B)} = 25$. From (44)–(47), we have $\bar{S}^{(B)} = 2$ms, $\sigma_S^{(B)} = 0.74$ms, $\bar{W}^{(q)(B)} = 48$ms and $\sigma_{W^{(q)}}^{(B)} = 48$ms. It is clear that for this user the queueing delay is the dominant component of the total delay. This can also be seen from Fig. 2. Therefore, the cumulative distribution function (CDF) of $W^{(B)}$, i.e., $\Pr\{W^{(B)} \leq t\}$, can be very accurately approximated by the CDF of $W^{(q)(B)}$. Hence, we can use (56) to numerically compute the CDF of the queueing delay. This CDF is plotted in Fig. 8. It is seen from the figure that about 63% of the times, the delay experienced by a packet is less than the average delay bound and 85% of the times, the delay is less than twice the average delay.

# VIII. Conclusions

We have studied the cross-layer problem of QoS-constrained power and rate control in wireless networks using a game-theoretic framework. We have proposed a non-cooperative game in which users seek to choose their transmit powers and rates in such a way as to maximize their utilities and at the same time satisfy their QoS requirements. The utility function considered here measures the number of reliable bits transmitted per Joule of energy consumed. The QoS requirements for a user consist of the average source rate and an upper bound on the average delay where the delay includes both transmission and queueing delays. We have derived the Nash equilibrium solution for the proposed game and obtained a closed-form solution for the user's utility at equilibrium. Using this framework, we have studied the tradeoffs among throughput, delay, network capacity and energy efficiency, and have shown that presence of users with stringent QoS requirements results in significant reductions in network capacity and energy efficiency. The delay performance of users at Nash equilibrium are also analyzed.

# Appendix

## Proof of Proposition 3

Given $P_1(s) = \frac{(1-\rho)\lambda(e^{s\tau}-1)}{s(e^{s\tau}-1+f)}$, we can use inverse Laplace transform to write

$$p_1(t) = \lim_{R\to\infty} \frac{1}{2\pi i} \int_{\sigma-iR}^{\sigma+iR} P_1(s) e^{st} ds.$$

Using the residue theorem and contour integration from complex analysis [20], we have

$$p_1(t) = \sum_k \text{Res}\left[P_1(s)e^{st}, s_k^*\right]$$

where $s_k^* = \frac{1}{\tau}\left[\ln(1-f) + 2\pi i k\right]$.

If we let $a = \ln(1-f)$, then we have

$$p_1(t) = (1-\rho)\lambda \sum_{k=-\infty}^{\infty} \frac{(e^a - 1)e^{at/\tau + 2\pi i k t/\tau}}{(1-f)(a+2\pi i k)}.$$

For convenience, let us define $x = t/\tau$ and notice that $x \geq 0$ since the queueing delay is non-negative. Then, we can write

$$p_1(t) = -f(1-\rho)\lambda(1-f)^{x-1} \sum_{k=-\infty}^{\infty} \frac{e^{2\pi i k x}}{a + 2\pi i k} = -f(1-\rho)\lambda(1-f)^{x-1} \sum_{k=-\infty}^{\infty} \frac{(a - 2\pi i k)e^{2\pi i k x}}{a^2 + 4\pi^2 k^2}.$$

Define $h(x) = \sum_{k=-\infty}^{\infty} \frac{(a-2\pi i k)e^{2\pi i k x}}{a^2+4\pi^2k^2}$. Then, we have

$$p_1(t) = -f(1-\rho)\lambda(1-f)^{x-1} h(x). \tag{57}$$

We can rewrite $h(x)$ as

$$h(x) = \frac{1}{2\pi}\left[\sum_{k=-\infty}^{\infty} \frac{b e^{2\pi i k x}}{b^2 + k^2} - i \sum_{k=-\infty}^{\infty} \frac{k e^{2\pi i k x}}{b^2 + k^2}\right]$$

where $b = \frac{a}{2\pi}$. We can equivalently write $h(x)$ as

$$h(x) = \frac{1}{2\pi b} + \frac{b}{\pi}\left[\sum_{k=1}^{\infty} \frac{\cos 2\pi kx}{b^2+k^2} + \sum_{k=1}^{\infty} \frac{k \sin 2\pi kx}{b^2+k^2}\right]. \quad (58)$$

Now, given the following Fourier series expansions [21]

$$\sum_{k=1}^{\infty} \frac{\cos ky}{b^2+k^2} = \frac{\pi}{2b}\frac{e^{b(\pi-y)}+e^{-b(\pi-y)}}{e^{b\pi}-e^{-b\pi}} - \frac{1}{2b^2} \quad \text{for} \quad 0 < y < 2\pi$$

and

$$\sum_{k=1}^{\infty} \frac{k\sin ky}{b^2+k^2} = \frac{\pi}{2}\frac{e^{b(\pi-y)}-e^{-b(\pi-y)}}{e^{b\pi}-e^{-b\pi}} \quad \text{for} \quad 0 < y < 2\pi$$

and after some manipulations, $h(x)$ becomes

$$h(x) = \frac{e^{-2\pi b(x-n)}}{1-e^{-2\pi b}} \quad \text{for} \;\; n < x < n+1.$$

Remembering that $a = \ln(1-f)$, we can simplify $h(x)$ to get

$$h(x) = \frac{(1-f)(1-f)^{-(x-n)}}{-f} \quad \text{for} \;\; n < x < n+1. \quad (59)$$

Since $p_1(t) = -f(1-\rho)\lambda(1-f)^{x-1}h(x)$ and recalling that $x = \frac{t}{\tau}$, we get

$$p_1(t) = \lambda(1-\rho)(1-f)^n \quad \text{for} \;\; n\tau < t < (n+1)\tau$$

or equivalently

$$p_1(t) = \lambda(1-\rho)(1-f)^{\lfloor \frac{t}{\tau} \rfloor}.$$

This completes the proof.

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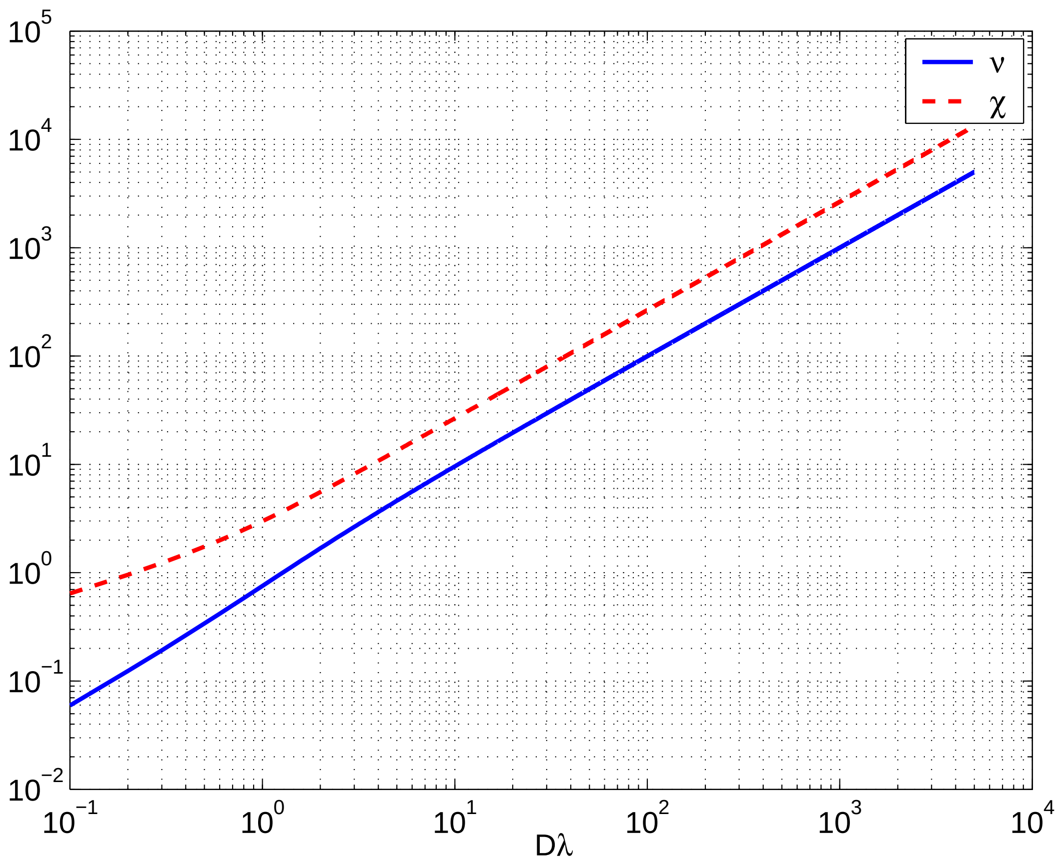


Fig. 2. Plots of $\nu = \frac{\bar{W}^{(q)}}{\bar{S}}$ and $\chi = \sqrt{\frac{\sigma^2_{W^{(q)}}}{\sigma^2_S}}$ as a function of $d = D\lambda$. $\lambda$ is the average source rate in packets per second and $D$ is the average delay bound in seconds.

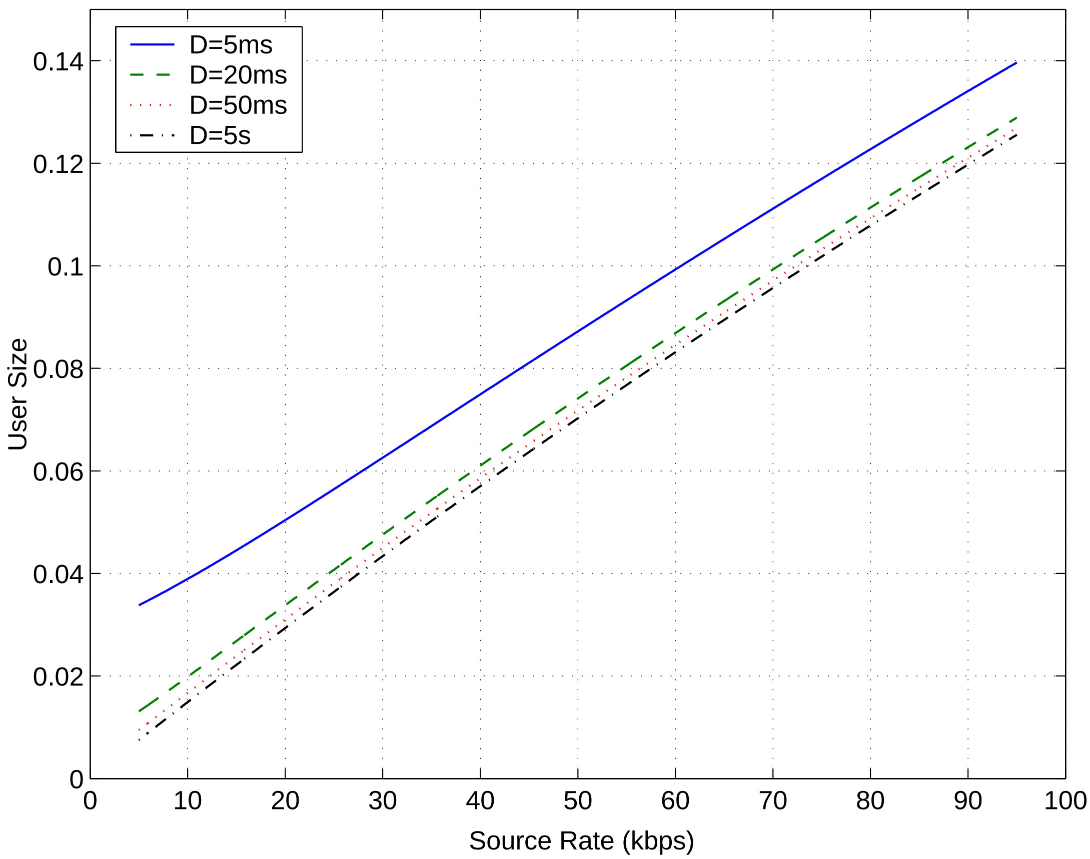


Fig. 3. User size, $\Phi^*$, as a function of source rate for different delay requirements.

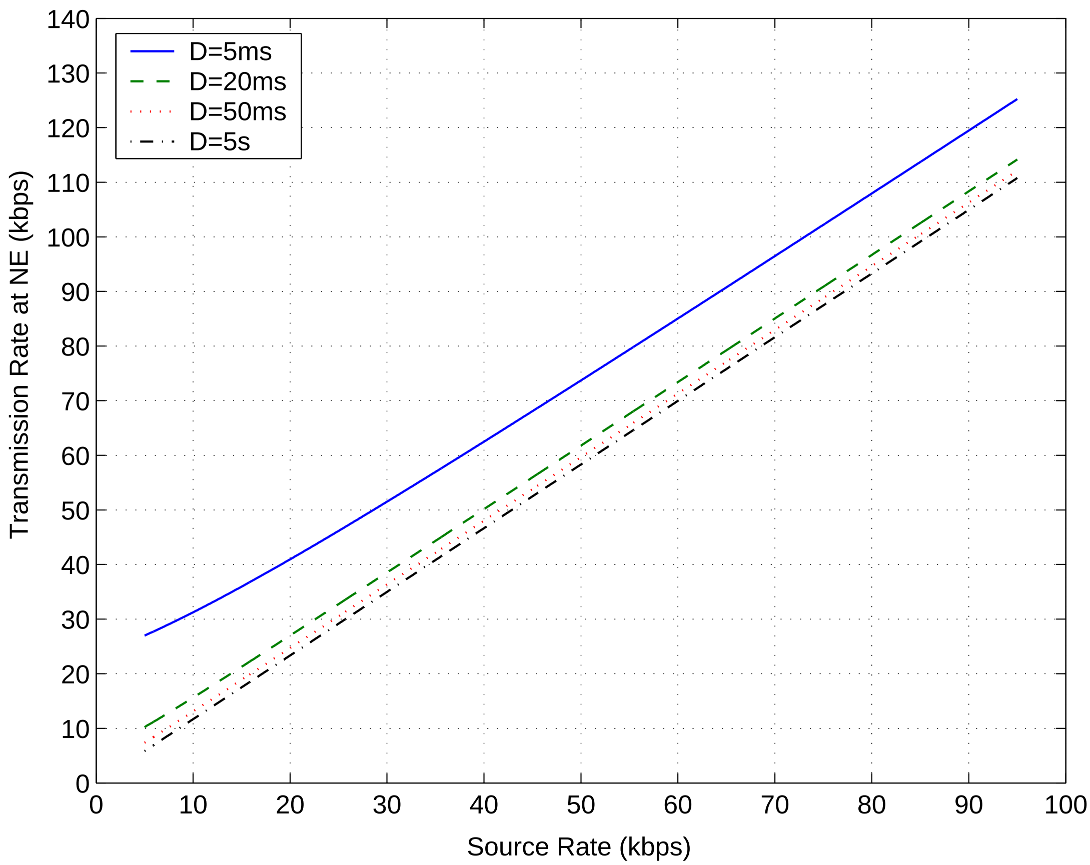


Fig. 4. User transmission rate, $\Omega^*$, as a function of source rate for different delay requirements.

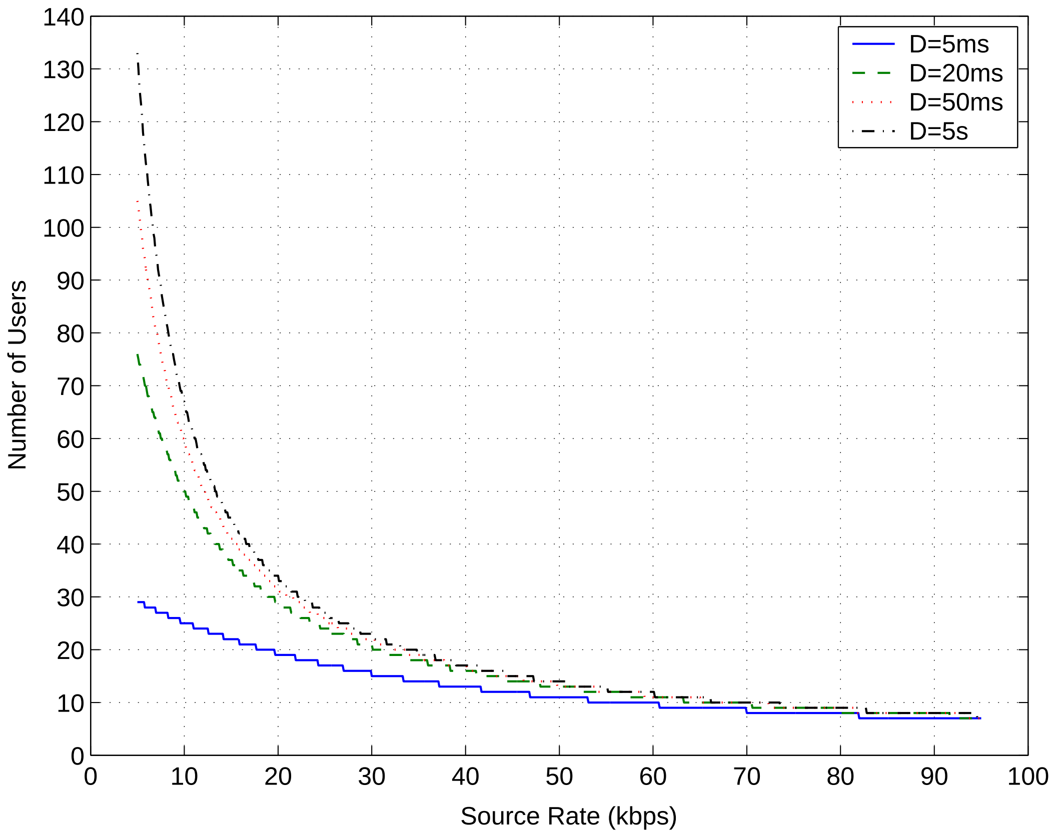


Fig. 5. Network capacity as a function of source rate for different delay requirements. Network capacity is defined as the maximum number of users whose quality of service requirements can be accommodated.

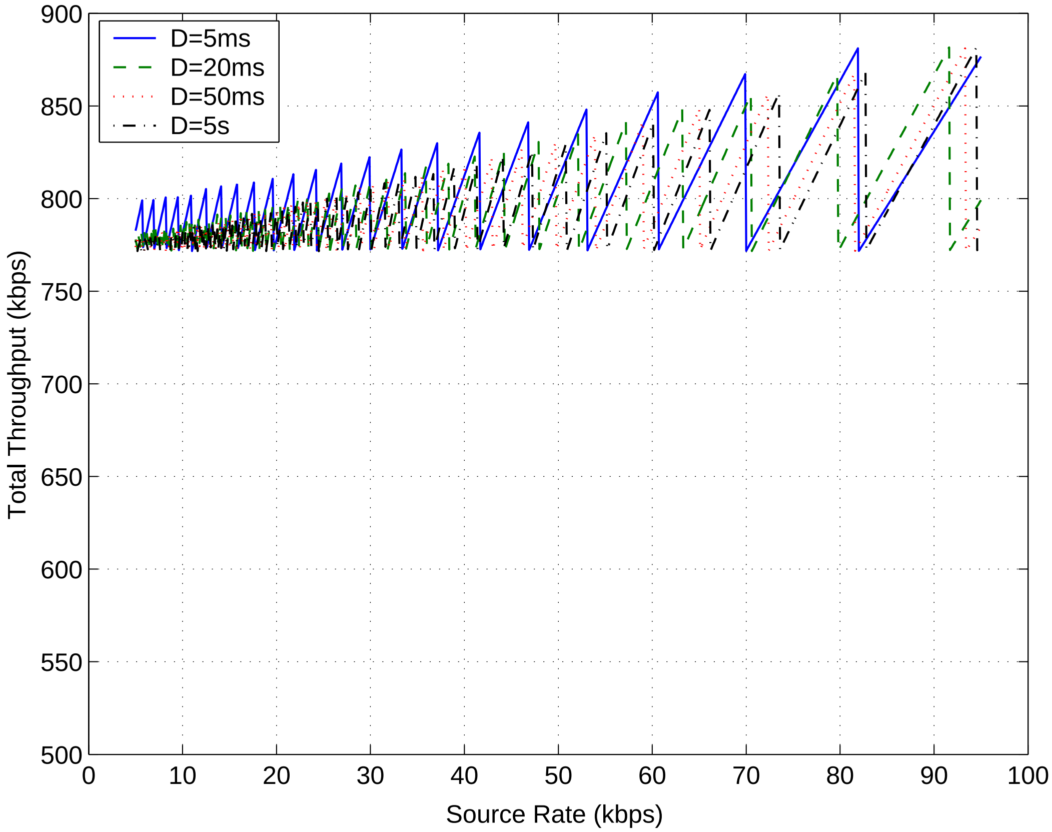


Fig. 6. Total throughput as a function of source rate for different delay requirements.

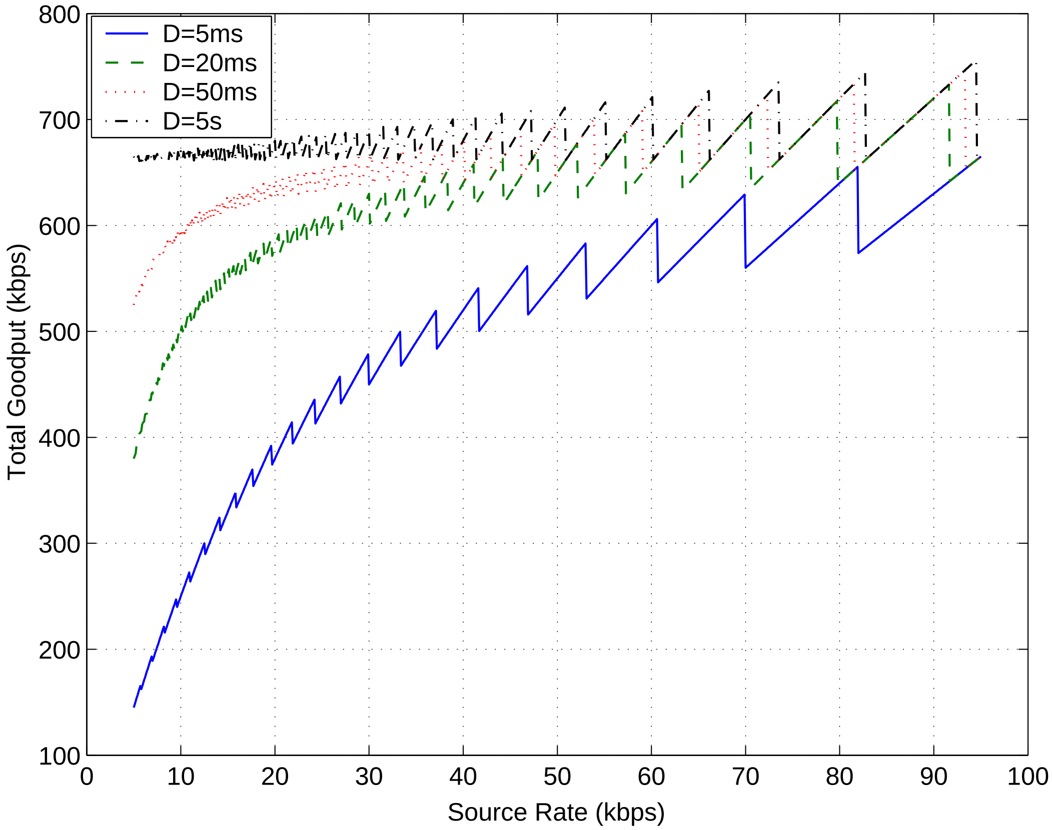


Fig. 7. Total goodput as a function of source rate for different delay requirements.

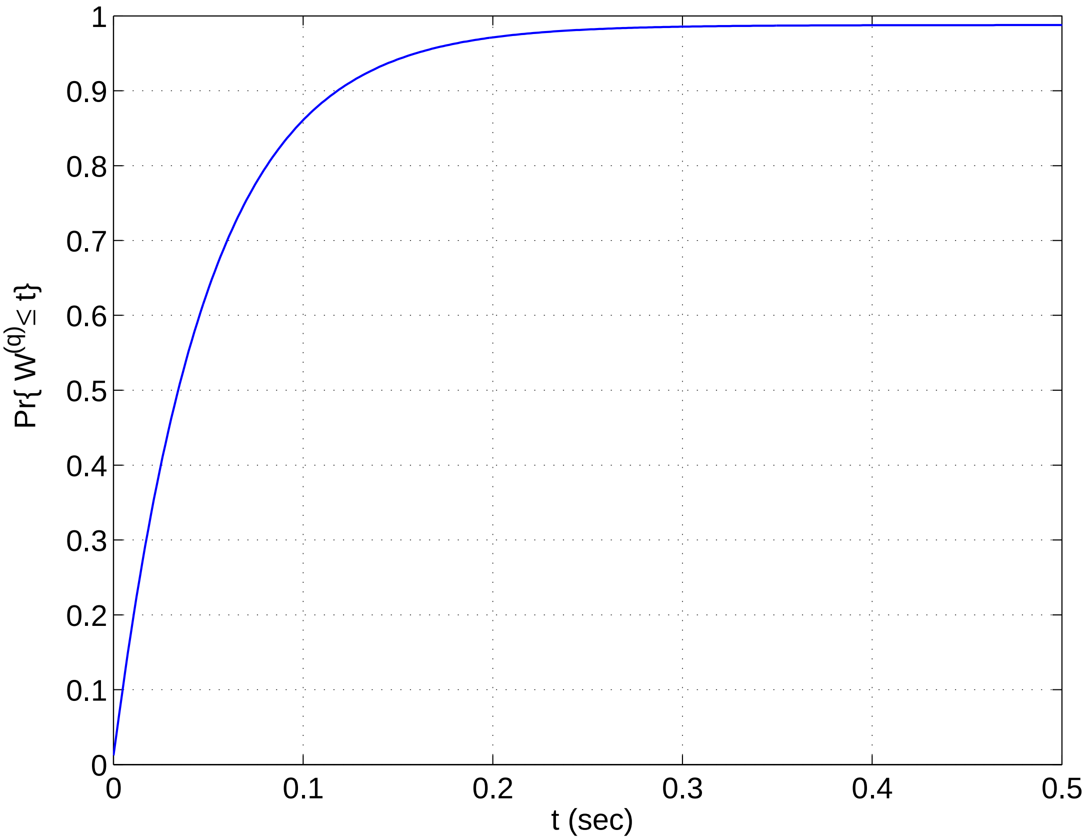


Fig. 8. Cumulative distribution function of the queueing delay for a user with a source rate of 50kbps and an average delay of 5ms.