UPGRADE OF THE GSP GYROKINETIC CODE

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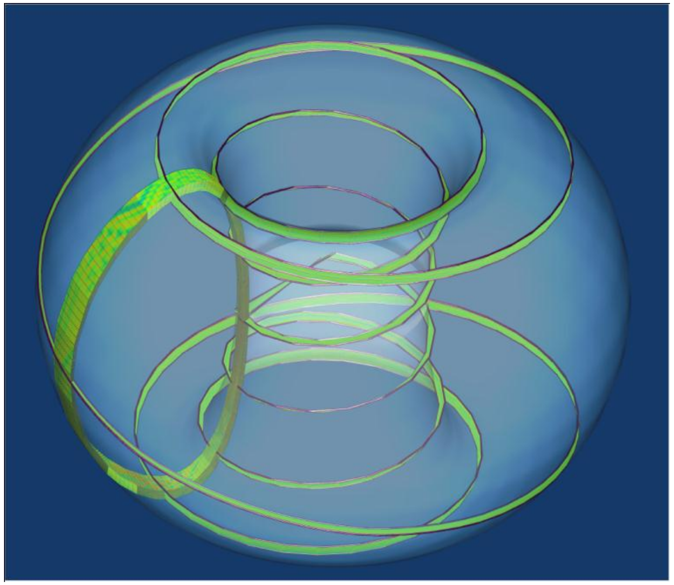
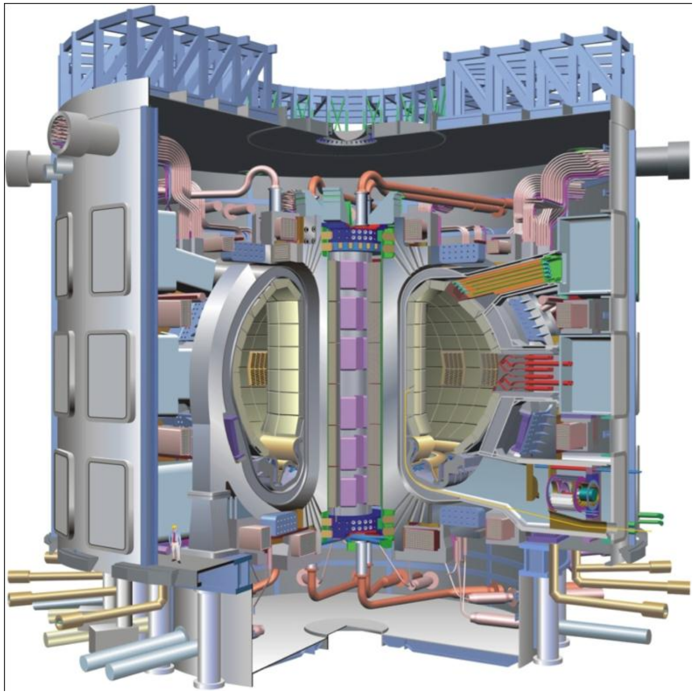
Abstract:

Simulations of turbulent plasma in a strong magnetic field can take advantage of the gyrokinetic approximation, the result of which is a closed set of equations that can be solved numerically. An existing code, GSP, uses a novel and highly efficient solution method. In this project, we will seek to change the velocity-space representation in GSP. This transformation will simplify the inclusion of a new collision operator and make the algorithm more suitable for simulations of turbulence in Tokamak plasmas, while retaining the efficiency and accuracy of the original code.

Motivation

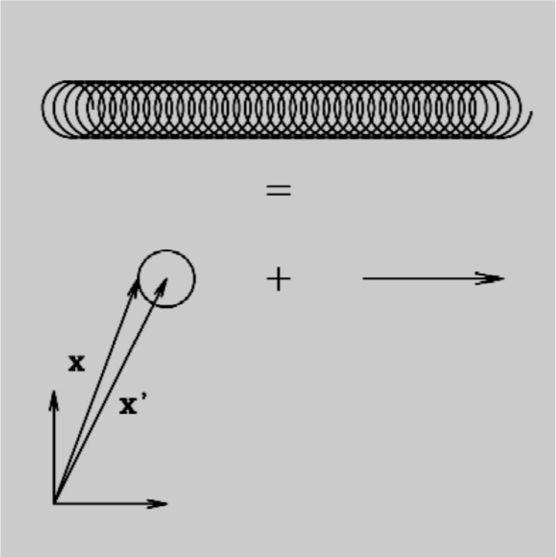
- Contained thermonuclear fusion has been a goal of plasma physics for several decades
- An understanding of turbulent transport in a magnetized plasma is critical, and simulations play an important role





Basic Physics

- Charged particles in a uniform magnetic field move in a helix
- Other effects (field gradients, curvature, electric field, etc.) will cause the helical motion to "drift"
- In a strong enough magnetic field, these drifts will be very slow compared to the circular motion of the charge



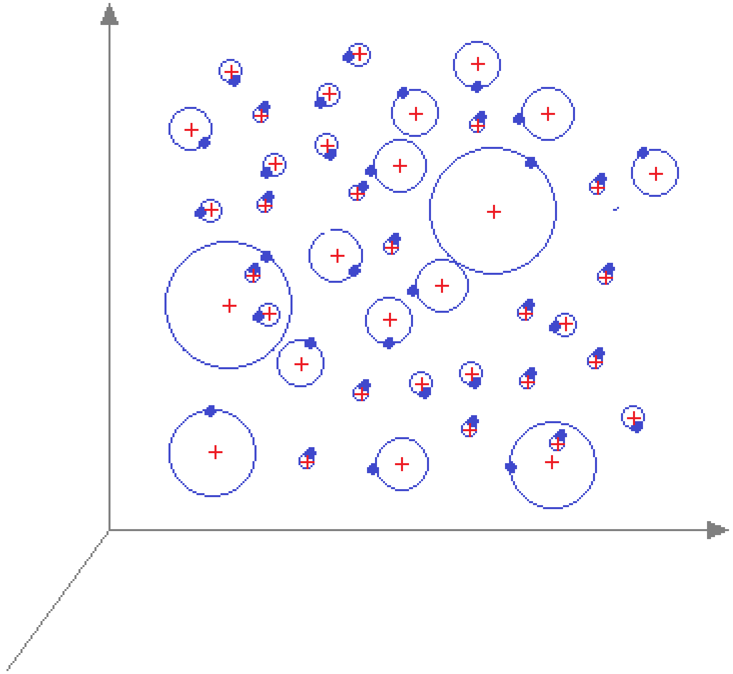
Basic Physics

- There is a rigorous theoretical framework for treating a collection of many interacting particles statistically

- Physical number of particles: $N \sim 10^{23}$

- The *gyrokinetic approximation* allows for closure of the theory based on averaging the oscillatory motion, treating the charge as a ring

- This reduces the problem to two independent components of velocity instead of three



Form of the Gyrokinetic Equation

$$\frac{\partial f}{\partial t} + \mathbf{A} \cdot \nabla_{\mathbf{R}} f + B \frac{\partial f}{\partial U} + C \frac{\partial f}{\partial V} = S[f]$$

- 6D integro-differential equation
- $\mathbf{A}$, B, C are, in general, functions of $\mathbf{R}$, t, U, V,
  and the *local potential* $\chi = \chi[f]$
- $S[f]$ includes an integrals of *f* such as the *collision operator* and the sources for Maxwell's equations

Method of Characteristics

$$\frac{\partial f}{\partial t} + \mathbf{A} \cdot \nabla_{\mathbf{R}} f + B \frac{\partial f}{\partial U} + C \frac{\partial f}{\partial V} = S[f]$$

- Define trajectories such that: $\mathbf{A} = \frac{d\mathbf{R}}{dt}, B = \frac{dU}{dt}, C = \frac{dV}{dt}$ and solve the integral *ODEs* $\frac{df}{dt} = S[f]$ along these curves
- **R** is the *gyrocenter position*, not the oscillating actual position of the physical particle
- Number of particles in simulation: $N \sim 10^9$

Method of Characteristics

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \nabla_{\mathbf{R}} f + \frac{dU}{dt}\frac{\partial f}{\partial U} + \frac{dV}{dt}\frac{\partial f}{\partial V} = S[f]$$

- Simplify characteristic curves by choosing ***constants of motion*** so that: $\frac{dU}{dt} = \frac{dV}{dt} = 0$
- Appropriate choices are:
  - Energy $E = \frac{1}{2} mv^2 + e\phi$
  - Magnetic Moment $\mu = \frac{1}{2}mv_\perp^2 / B_0$

Algorithm

1. Initialize particles
   - Sample velocities from Maxwellian
   - Uniform random distribution of positions
   - Initialize fields
2. Solve “ODEs” for each particle
   - Advance particles along characteristic trajectories
3. Use new df to solve for new potentials in Fourier space
   - Fourier transform back and smooth onto grid
4. Repeat Steps 2-3
5. Use solution to calculate evolved fluxes and densities

Code: **GYRO**

Authors: Jeff Candy and Ron Waltz

GSP Code

- Written by: Ingmar Broemstrup
- Language: FORTRAN 95
- Platform: Linux desktop or array of processors
- ~3000 lines of code

- **Changes to make:**
  - Add diagnostic tools to test accuracy
  - Change velocity coordinates to E, μ
  - Allow for gradients in equilibrium magnetic field
  - Introduce collision operator from Abel, et al. (2008)
  - (Change spatial coordinates to toroidal geometry)

Predictable Problems

- Insufficient understanding on part of the student to debug changes to GSP code by May
- Changes to the code do not result in faster or more accurate modeling

Validation and Testing

- Compare to original GSP code
  - Ensure coordinate change alone does not affect result
- Performance benchmarks:
  - General speed compared to original GSP code
  - Verify efficient parallelization scaling
- Compare against analytical result
- Compare results against GS2 or AstroGK

Databases

- GSP Code:

  http://gyrokinetics.svn.sourceforge.net/viewvc/gyrokinetics/gsp/

- Article with analytic result:

  Ricci, et al, "Gyrokinetic linear theory of the entropy mode in a Z pinch." *Physics of Plasmas*, **13**: 062102

- Article with computational benchmark:
  - Dimits, et al, "Comparisons and physics basis of tokamak transport models and turbulence simulations." *Physics of Plasmas*, **7**: 969

	Timeline	**Milestone**
Phase I	September – October	Transformed GK equation derived and algorithm understood
Phase II	October – January	Changes coded and debugged
Phase III	February – April	New code validated, tested, and benchmarked
Phase IV	April – May	Results organized, presentation prepared and given

Deliverables:

- Updated GSP source code and sample input files
- Results compared to databases
- Mid-year progress report
- Final presentation

Bibliography

- Howes, et al. “Astrophysical gyrokinetics: basic equations and linear theory.” *The Astrophysical Journal*, **651**: 590 (2006)
- Abel, et al. “Linearized model Fokker-Planck collision operators for gyrokinetic simulations. 1. Theory.” *Physics of Plasmas*, **15**:122509 (2008)
- Antonsen and Lane, “Kinetic equations for low frequency instabilities in inhomogeneous plasmas.” *Physics of Fluids*, **23**:1205 (1980)
- Broemstrup, “Advanced Lagrangian Simulations Algorithms for Magnetized Plasma Turbulence.” PhD Thesis, University of Maryland (2008)
- Catto, “Linearized gyro-kinetics.” *Plasma Physics*, **20**:719 (1978)
- Frieman and Chen, “Nonlinear gyrokinetic equations for low-frequency electromagnetic waves in general plasma equilibria.” *Physics of Fluids*, **25**:502 (1982)

Questions?

Maxwell's Equations

- **Poisson's Equation:** $$\nabla^2\phi = -4\pi\left(q_i n_i + q_e n_e\right)$$

$$n_i = \int \delta f_1\left(\mathbf{r}, \mathbf{v}, t\right) d^3\mathbf{v}$$

$$n_e = n_0 \frac{e\phi}{T_e}$$

- **Ampere's Law:** $$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}$$

$$\mathbf{J} = e n_i \overline{\mathbf{v}}_i - e n_e \overline{\mathbf{v}}_e$$

$$n_i \overline{\mathbf{v}}_i = \int \delta f_1 \mathbf{v} d^3\mathbf{v}$$

Collision Operator

- Generally dissipative

- Integral operator
  - Effect of collision depends on *relative* velocity between colliding particles.
  - Since the collision partner has its own distribution of velocities, we must integrate over its distribution
  - The function we need to solve for is inside the integrand

- There are physical forms based on various assumptions, and forms that easily computed
  - Boltzmann: Short-duration, binary collision. Applies to cases dominated by collisions with neutral atoms, not to a well-ionized plasma.
  - Fokker-Planck/Landau: Small perturbation to velocity, many-body interaction.
  - Krook: Simple, but non-physical
  - Abel, et al.: Computationally efficient, satisfies physical conservation properties

Gyroaveraging Fourier Transforms

$$\left\langle e^{i\mathbf{k}\cdot\mathbf{r}} \right\rangle_{\mathbf{R}} = \frac{1}{2\pi}\int e^{i\mathbf{k}\cdot(\mathbf{R}+\rho)} d\theta = \frac{1}{2\pi} e^{i\mathbf{k}\cdot\mathbf{R}} \int e^{-i\mathbf{k}\cdot\frac{\mathbf{v}}{\Omega}} d\theta$$

$$= \frac{1}{2\pi} e^{i\mathbf{k}\cdot\mathbf{R}} \int e^{-i\frac{k_{perp} v_{perp}}{\Omega}\cos\theta} d\theta$$

$$= e^{i\mathbf{k}\cdot\mathbf{R}} J_0\left(\frac{k_{perp} v_{perp}}{\Omega}\right)$$

Gyrokinetic motion