Using Genetic Algorithms to solve the Minimum Labeling Spanning Tree Problem

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Abstract: Cellular Genetic Algorithms (CGAs) have shown themselves to be very powerful tools for combinatorial optimization. Through this project I hope to investigate CGAs, develop a parallel implementation of a CGA, use these techniques on the Minimum Labeling Spanning Tree Problem, and compare results with other heuristics.

Introduction to MLST

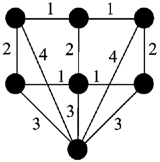
- First proposed in 1996 [Chang:1996]- variant on minimum weight spanning tree

- Connected Graph - set of vertices and edges.
- Each edge has a color
- Find the smallest set of colors which gives a connected sub-graph

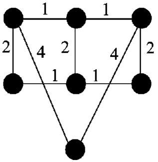
An example of a labelled spanning tree, and some feasible solutions [Xiong:2005]

Question: What is the smallest set of colors which induces a connected (sub-) graph?

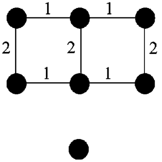
Complete Graph G



Subgraph induced by {1, 2, 4} - Connected



Subgraph induced by {1, 2} - Not Connected



Introduction to MLST

- First proposed in 1996 [Chang:1996]- variant on minimum weight spanning tree
- Shown to be NP-complete
- Two heuristics and an exhaustive search proposed in the original paper - heuristics achieved moderate success

Introduction to Genetic Algorithms (GAs)

- Evolutionary-inspired heuristic for optimization problems

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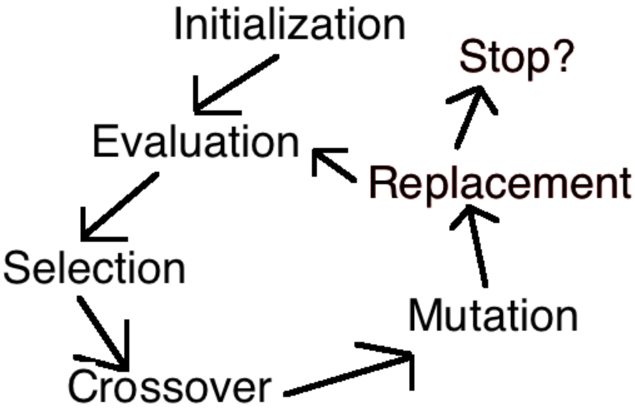
- Evolutionary-inspired heuristic for optimization problems
- Population = set of solutions
- Select, Breed, Replace
- Advantages:
  - Flexible and adaptable
  - Robust performance at global search
  - Simple to parallelize

Key steps in a Genetic Algorithm



One Parameter GA for MLST - Serial

- From Xiong, 2005
- Designed to be simple - no fine tuning
- One parameter - p, population size
- Representation: List of labels
- Gene: Label in the list

Step 1: Initialization

- Create first generation of individuals - viable, varied

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- Initialization from Xiong:2005:
  For each individual in population:
      Individual = {}
      While Individual Is Not Viable:
          Individual.AddRandomColor()

Step 2: Evaluation

- Defined by problem
- For some problems can be extremely time consuming
- Multiple criteria
  $\rightarrow$Penalty functions?

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  $\mathrm{Eval}(T) = \mathrm{len}(T)$

Step 3: Selection

- How? Random, Sweep, ...
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- Selection in Xiong:2005;
  for j = 1:Size(Population)
  Offspring(j) = Breed(Parent(j), parent(($j + k$) mod p))
  (where k is the generation number)

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- Choose genes randomly? Follow order (pick 'strongest' genes first)?

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- Crossover in Xiong:2005:
  $S =$ Union of genes (colors) from both parents
  Sort(S) %According to frequency of labels in Graph
  $T = \{\}$
  while T Is Not Viable:
      T.AddLabel(NextLabel(S))
  return T

Crossover operator

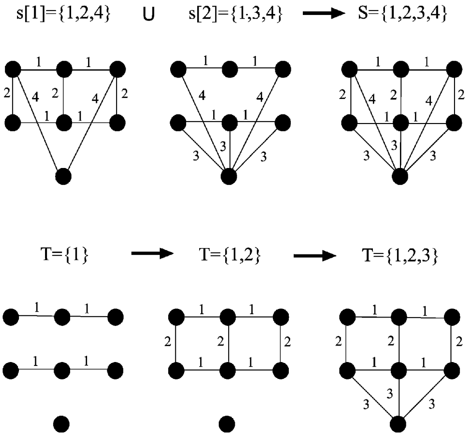

Figure: The crossover operator used in Xiong's GA [Xiong:2005]

Step 5: Mutation

- Introduce new genetic material
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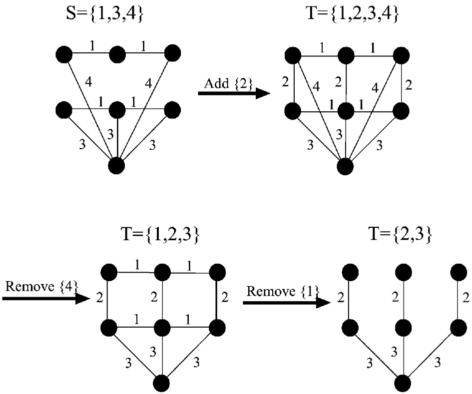
- Mutation in Xiong:2005 (100% chance of mutation):

```
T.AddRandomColor
Sort(T) %According to frequency of labels in Graph
For Label in T(-1:-1:): %Reverse iterate
    T.Remove(Label)
    if T Is Not Viable:
       if T.Add(Label)
return T
```

Mutation operator



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Step 6: Replacement

- Find new generation from strongest offspring and parents
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- Replace parents where warranted
- Replacement in Xiong:2005:
  If Eval(Offspring)<Eval(Parent):
      Parent.Replace(Offspring)

Step 7: Stopping Conditions

- Generations count/computational time
- Population Stagnant

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- Population Stagnant
- Stopping Condition in Xiong:2005: Do p generations

GA improvements

- Improve Crossover/Mutation operators?
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- Favor retention of mutated genes?
- Keep equally good offspring?
- Divide up population space - promote diversity

3 Different types of GA

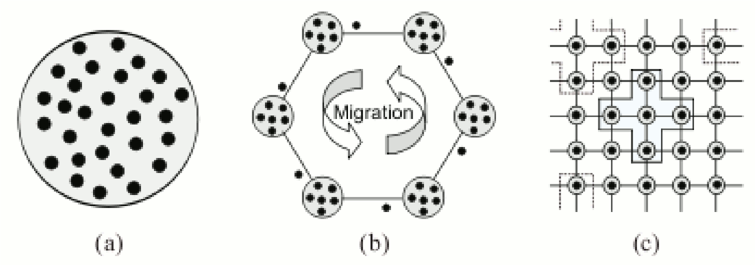

Figure: Three different types of GAs showing interaction between individuals (black dots) in the population. a)Panmictic b) Distributed c) Cellular [Alba:2008]

Genetic Algorithm $->$ Cellular Genetic Algorithm

- Modify Selection operator- limit to neighborhood on grid

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- Modify Selection operator- limit to neighborhood on grid
- Arrangement of entire population space
- Neighborhood size?
- Choosing within neighborhood:
  - Step through neighborhood
  - Randomly choose one
  - Pick 'strongest' neighbor?

Serial Cellular Genetic Algorithm $- >$ Parallel Cellular Genetic Algorithm

- Why?
  - Speedup
  - Larger Problems

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- Allocate nodes to separate processors
- Master-slave vs. direct communication

Parallel Structures

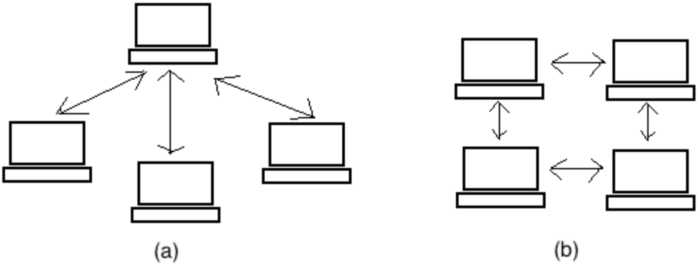

Figure: Different approaches to parallel programming. (a) Master/Slave configuration and (b) Inter processor communication

Cellular Genetic Algorithm $->$ Parallel Cellular Genetic Algorithm

- Why?
  - Speedup
  - Larger Problems
- Allocate nodes to separate processors
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- Lock nodes when in use. Queues?
- Synchronous (simultaneous) vs asynchronous

Hardware/Software/Databases

Language - C++ with MPI (Message Passing Interface)
Hardware - array of processors at UMD
Database: Randomly generated labeled spanning trees

Validation/Testing

- Comparing my serial CGA with other heuristics and with global optimum (if known, e.g. through exhaustive search)
- Compare parallel results with serial CGA, ensure as expected (feasible, function in the right range)
- Calculate speedup of parallel vs. serial, asynchronous vs. synchronous [Fujimoto:2011, Vidal:2010, Drummond:2001, Groer:2010]

Schedule: Part I

Part 1: Creating my serial Cellular Genetic Algorithm
Tasks:

- Adding improvements to the Genetic Algorithm
- Modifying selection operator/imposing grid structure so becomes CGA

Timing: Sept - Oct 2010
Result: Competitive, efficient serial CGA code
Validation: Compare computational effort, results with other heuristics (e.g. GA from Xiong, 2005)

Schedule: Part 2

Part 2: Going parallel

Tasks:

- Initially converting to synchronous code - direct communication, locking nodes ...
- Converting synchronous code to asynchronous code

Timing: Nov 2010- Jan 2011

Result: Efficient, parallel, asynchronous CGA code using direct communication

Validation:

- Check results match serial code
- Check speed-up rate of synchronous code over serial code (hopefully equal to number of processors)
- Check speed-up of asynchronous code over synchronous code

Schedule: Part 3

Part 3: Fine tuning/Polishing

Tasks:

- Determine optimum parameters, neighborhood/population space arrangement etc.
- Further optimize code if possible

Timing: Feb 2011

Result: Efficient, competitive, parallel, asynchronous CGA code using direct communication

Validation: Compare with earlier version of algorithm/with other algorithms used in literature

Schedule: Part 4

Part 4: Running on massive array/Reporting

Tasks:

- Run on powerful array of processors
- Prepare final report/presentation

Timing: Mar 2011-

Result:

- Results for larger problems than attempted earlier (incl % optimal, speed-up results ...)
- Parallel, asynchronous, competitive Cellular Genetic Algorithm code for the MLST using direct processor-processor communication
- Final report/presentation

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