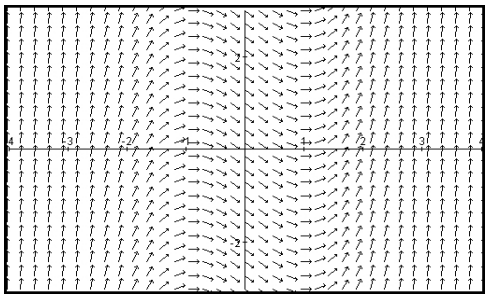
April 1, 2009 **Practice - Math 221 section 02xx Exam 2 [Shaw]**

No calculators allowed. You must show work at all times in order to receive full credit.

1. Which of the following is the differential equation whose direction field is represented in the figure to the right? *Give a one-sentence explanation why.*



   a. $y' = y + 1$

   b. $y' = t^2 - 1$

   c. $y' = e^y$

   d. $y' = y^2 + t^2$

2. Show that $f(t) = 3\cos(2t) + 5\sin(2t)$ is a solution to the initial value problem $2y'' + 8y = 0;\ y(\pi) = 3$.

3. Find $\displaystyle\int_{-\infty}^{1} 5te^{t^2}\,dt$. (If it diverges, write DNE.)

4. For each of the following initial value problems, find both the general solution and the particular solution, using any appropriate technique from class:

   (a) $ty' = \left(\dfrac{\ln(t)}{y}\right)^2,\ t > 0;\ y(1) = 2$

   (b) $y' + e^t y = e^t;\ y(0) = 2$

5. When buying a car, Blake took out a loan of $\$15,000$, with an annual interest rate of 10%, compounded continuously. If he pays off his loan with continuous payments at a rate of $\$2,500$ per year, how long before he pays off the car? (You do not have to simplify your answer.)

6. At UMd this semester, there are 1200 freshman, and mono is spreading rampantly throughout the class at a *monthly* rate which is proportional to the number of students who have it times the number who don't.

   (a) Set up a differential equation which is satisfied by $P(t)$ = the number of freshman students who have mono at time t, *measured in months*. (Note that your equation will have an unknown quantity k.)

   (b) Suppose that when there 300 students with mono, the growth rate of the mono infection is 150 students *per month*. Use this data to solve for k.

   (c) How many people have mono when the rate of spread is the highest?

   (d) Using your equation from part *a.* and your k-value from b., on one set of axes, sketch the yz-graph, and on another, a graph of solutions corresponding to $P(0) = 30$ and $P(0) = 800$.

7. Consider the IVP $y' = 2y - 3t;\ y(0) = 2$.

   (a) Let $f(t)$ be the solution to the IVP. Use Euler's method with $n = 3$ on $2 \le t \le 3$ to estimate $f(1)$.

   (b) The particular solution to the IVP is $f(t) = \dfrac{3}{2}t + \dfrac{3}{4} - \dfrac{15}{4}e^{2t}$. What is the error of the estimate you got in part *a.*? You do not have to simplify your answer.