

April 24, 2009

Practice - Math 221 section 02xx Exam 3 [Shaw]

No calculators allowed. You must show work at all times in order to receive full credit.

1. (a) Use the Newton-Raphson method with $n = 2$ to estimate a value for $\sqrt{10}$. You do not need to simplify your answer. *Hint: Your x_0 should be 3.*
(b) In order to estimate a value for $\sin\left(\frac{11\pi}{20}\right)$, do the following:
 - i. Find the second Taylor polynomial $p_2(x)$ of $f(x) = \sin(x)$ about $x = \frac{\pi}{2}$.
 - ii. Find $p_2\left(\frac{11\pi}{20}\right)$. You do not need to simplify your answer.

2. (a) Using a geometric infinite series, represent $1.23\overline{23}$ as a ratio of integers (either a fraction or a mixed number).
(b) Find the sum of the following geometric series; if the series is divergent, explain why:
$$\sum_{n=0}^{\infty} 2\left(\frac{2}{7}\right)^n$$

3. Determine whether $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin\left(\frac{1}{n}\right)$ is convergent. (You do not have to find the sum if it is convergent.)

4. (a) Using the Taylor series about 0 for $\frac{1}{1-x}$, find the Taylor series about 0 for $\frac{x^2}{1+x^3}$. Show at least the first four terms of the series.
(b) Suppose the Taylor series about $x = 0$ for the function $g(x)$ is:

$$1 - \frac{1}{2!}x^2 + \frac{1}{3!}x^4 - \frac{1}{4!}x^6 + \frac{1}{5!}x^8 - \dots$$

Find the Taylor series about $x = 0$ for $G(x)$, where $G(x) = \int 2g(x) dx$, $G(0) = 2$.

5. The Mathematics department does a survey about the number of job offers that its recent PhD graduates get. The results are displayed in the following frequency table:

0 offers	12
1 offer	11
2 offers	5
3 offers	2
> 3 offers	0

- (a) Find the expected number of job offers for a recent Mathematics PhD.
- (b) Set up a calculation to find the variance, *but do not solve*.
- (c) Describe how, once you had the variance, you could find the standard deviation.