Final Exam Review

Math 221 Sec. 02**

C. Shaw

May 8, 2009

Facts

- Review: Wednesday, 2-4pm, KEY 0106
- Exam: Thursday, 1:30–3:30
  Ilya's class: SPH 1312
  Russ's class: HJP 2242
- You may not use your cell phone as a watch.
- Be sure to arrive early, no extra time.

Outline

Example 1: Spring 2006, prob. 1

Find the equation for the tangent line to the function:

$$f(x) = (x+1)\cos(x^2 + 2x)$$

at the point where $x = 0$.

Example 2: Spring 2007, prob. 1a

Calculate:

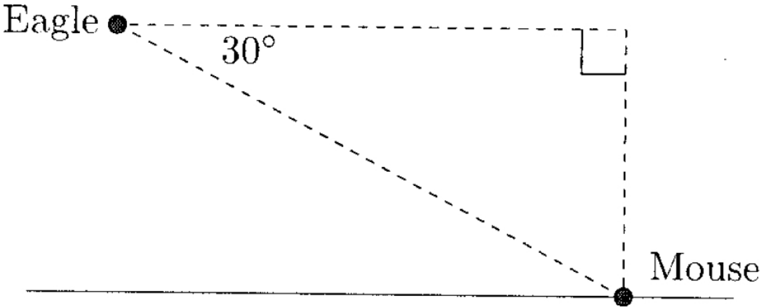
$$\frac{d}{dt}\ln(6t)\cos^{10}(7t)$$

Example 3: Fall 2007, prob. 1a

Find an angle t with $\pi < t < 2\pi$ with $cos(t) = \cos\left(\frac{16\pi}{7}\right)$.

Example 4: Fall 2007, prob. 1b

An eagle flying at an altitude of 1000 meters sees a mouse on the ground at an angle of 30° as shown. How far is the mouse from the eagle? **Simplify.**



Example 5: Made-up, just for you, on this day, by me

Find and classify the critical point(s) for the function $y = \cos^2(x)$ from $-\frac{\pi}{4} < x < \frac{\pi}{4}$.

Outline

Example 1: Spring 2006, prob. 2

Use the midpoint rule M, the trapezoidal rule T, and Simpson's rule S with two subintervals to approximate the integral:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(x)\, dx$$

Example 2: Spring 2006, prob. 3a

Compute the integral:

$$\int x \cos(5x + 1)\, dx$$

Example 3: Spring 2006, prob. 3b

Compute the integral:

$$\int_0^{\infty} \frac{3}{(4x+5)^2}\,dx$$

Example 4: Fall 2006, prob. 2a

Compute the integral:

$$\int x^5 \ln(x)\, dx$$

Example 5: Fall 2006, prob. 2b

Compute the integral:

$$\int_1^\infty \frac{x+3}{(x^2+6x+4)^2}\,dx$$

Example 6: Made-up, just for you, on this day, by me

Find the area under the function $y = 4x\sec^2(x^2)$, on the interval $[0, \frac{\sqrt{\pi}}{2}]$.

Example 7: Spring 2007, prob. 2b

Use the Trapezoidal rule with 4 subintervals to find an approximation to:

$$\int_1^5 x^2\,dx$$

Example 8: Spring 2008, prob. 1aii

Integrate:

$$\int \frac{(\ln(x))^2}{x}\,dx$$

Outline

Example 1: Fall 2006, prob. 4

Consider the differential equation $y' = (y-1)(y-3)$.

(a) Find the constant solutions.

(b) Sketch the solution with initial condition $y(0) = 0.25$.

(c) Find an approximate value for $y(1000000)$ when $y(0) = 2$.

Example 2: Spring 2006, prob. 4

Solve the differential equation $y' + \frac{1}{4}y = 4$ with initial condition $y(0) = 0$ and also with $y(0) = 16$.

Example 3: Spring 2007, prob. 3a

Find all solutions to the differential equation:

$$y' = \frac{e^t}{y^2}$$

Example 4: Spring 2006, prob. 5b-c

(b) A patient receives a continuous infusion of a drug into the bloodstream, at the rate of 4 mg per day. The patient's body eliminates the drug at the daily rate of 25% of the drug present in the system. Let $y = f(t)$ represent the amount of the drug present in the body at time t (with time measured in days). Set up the differential equation solved by y.

(c) Determine how many mg of the drug is in the bloodstream after a long time.

Example 5: Fall 2006, prob. 3b

In a certain forest, dead vegetation forms on one square centimeter of ground at a rate of 50 grams per year. The dead vegetation decomposes at a rate of 80% per year.

(i) Find a differential equation satisfied by the amount $y = f(t)$ of dead vegetation present at time t. Your differential equation should have the form $y' = ay + b$ for certain constants a and b.

(ii) Determine approximately how many grams of dead vegetation are present after many years.

Outline

Example 1: Spring 2006, prob. 6b

A patient receives 4 mg of a certain drug, once a day, at the same time each day. In one full day, the patient's body eliminates 25% of the drug present in the system.

(i) Write an expression that gives the amount of the drug in the patient's body immediately after the third dose has been given (two days after the initial dose).

(ii) Estimate the approximate total amount of drug present in the patient's body after many weeks of treatment, immediately after a dose is given.

Example 2: Spring 2007, prob. 4a

Determine whether the series converges or diverges. If it converges, find the sum:

$$\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n$$

Example 3: Spring 2006, prob. 7a-b

(a) Compute the third Taylor polynomial for the function $f(x) = \ln(x)$ at $x = 1$.

(b) Find the coefficient to $(x-1)^{100}$ in the 100^{th} Taylor polynomial for $f(x) = \ln(x)$ at $x = 1$.

Example 4: Fall 2006, prob. 1a

For each of the following, find the Taylor series at $x = 0$ through the x^8 term:

(i) $f(x) = \frac{1}{1-x^4}$

(ii) $g(x) = \frac{1}{(1-x)^2}$

Example 5: Spring 2007, prob. 5a

Find the Taylor series around $x = 0$ for the function $f(x) = xe^{(x^2)}$. Show at least four non-zero terms.

Example 6: Spring 2007, prob. 5b

Find a 2^{nd} degree Taylor polynomial of $f(x)$ around $a = 9$ and use it to obtain an approximation of $\sqrt{8}$. You may leave your answer as a sum of fractions.

Example 7: Spring 2007, prob. 4b

Determine whether the series converges or diverges. If it converges you do not have to find the sum:

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$$

Outline

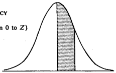
Example 1: Fall 2006, prob. 5a

Find the value of k such that $f(x) = kx^2$ is a probability density function for $0 \leq x \leq 2$.

Example 2: Fall 2007, prob. 8b

The amount of soda in a soda can coming off the production line is approximately normally distributed with a mean of 16oz and standard deviation of 0.5oz. What is the probability that a randomly chosen soda can will contain over 16.85oz of soda?

TABLE A 3
CUMULATIVE NORMAL FREQUENCY DISTRIBUTION
(area under standard normal curve from 0 to Z)



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000									

Example 3: Fall 2007, prob. 8a

Suppose a certain event has probability density function $f(x) = \frac{3}{26}x^2$ for $1 \leq x \leq 3$.

(i) Find $P(1 \leq X \leq 2)$.

(ii) Find $E(X)$.

(iii) Find $Var(X)$.

(iv) Find the cumulative distribution function $F(x)$.

Example 4: Spring 2006, prob. 10

Assume that the number X of typographicla errors per paeg of a certain newspapre is a Poison random variable and the probabilty is .5 that there are no errors on a on a page.

(a) What is the probabililty that a page has mor than 1 error?

(a) What is the avergae number of erros per page?

Example 5: Made-up, just for you, on this day, by me

At a fishhook factory, 1.5% of the barbed treble hooks that come off of the assembly line are missing a barb on one of the hooks. A quality control tester checks the hooks randomly for errors.

(a) What is the probability that the inspector finds exactly four good hooks in a row before finding a bad hook?

(b) What is the probability that the inspector finds at least four good hooks in a row without finding a bad hook?