Useful Properties of the Determinant

Let A, B be an $n \times n$ matrices.

1. $\det(AB) = \det A \det B$.

2. $\det A^T = \det A$.

3. If any two rows of A are equal, than $\det A = 0$.

4. $\det(cA) = c^n \det A$ for any constant c.

5. Multiplying any row by a constant multiplies the determinant by that constant. That is, for any constant c,

$$\det(A_{R_i \to cR_i}) = c \det A.$$

6. Interchanging any two rows of A changes the sign of $\det A$.

$$\det(A_{R_i \leftrightarrow R_j}) = -\det A$$

7. Adding a multiple of one row to another row does not change the determinant.

$$\det(A_{R_i \to R_i + cR_j}) = \det A.$$

Notes:

1. If A is invertible, it follows from Property 1 that $\det(A^{-1}) = \frac{1}{\det A}$.

2. **Theorem:** A is invertible if and only if $\det A \neq 0$.

3. **Theorem:** A is one-to-one and onto if and only if $\det A \neq 0$.

4. From Property 2, it follows that properties for *column* operations analogous to those for row operations stated above also hold.

5. You can compute the determinant using the formula of Definition 6.5 of Colley Section 1.6 (page 55).

6. If A is (upper or lower) triangular, then the determinant is the product of the diagonal entries. Using row operations (and keeping track of how they change the determinant according to the properties above), you can thus simplify the computation of the determinant, as in Colley 1.6 Exercise 27.