1. For each of the following, determine if it is a vector space or not, and give the rules of addition and scalar multiplication.

a. All functions $f$ from $\mathbb{R}$ to $\mathbb{R}$ which satisfy $f(0) \neq 0$.

b. All functions $f$ from $\mathbb{R}$ to $\mathbb{R}$ which satisfy $f(0) = 0$.

c. All functions $f$ from $\mathbb{R}$ to $\mathbb{R}$ which have a second derivative.

d. All polynomials of the form $p(x) = ax^2 + bx$, where $a, b$ are in $\mathbb{R}$.

e. All polynomials of the form $p(x) = x^2 + c$, where $c$ is in $\mathbb{R}$.

f. All solutions $y = y(x)$ to the differential equation $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = 0$.

g. All solutions $y = y(x)$ to the differential equation $\frac{dy}{dx} = \cos x$.

h. All matrices of the form

\[
\begin{pmatrix}
a & b \\
0 & c
\end{pmatrix}
\]

where $a, b, c$ are in $\mathbb{R}$.

2. The following is a vector space: All matrices of the form

\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}
\]

where $a, b, c, d$ are in $\mathbb{R}$. Using the 10 axioms, prove that it is a vector space. Find “vectors” (i.e., matrices) that span it.