1. a. Find a basis for the subspace of $\mathbb{R}^2$ spanned by the line 

$$x + y = 0$$

b. Find a basis for the vector space of all $2 \times 2$ upper triangular matrices.

c. Find a basis for the subspace of $\mathbb{R}^3$ spanned by the plane 

$$2x - 4y + 3z = 0$$

d. The set 

$$\beta = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}$$

is not a basis of $\mathbb{R}^4$. By removing any single vector and adding any single vector of your choice, change it into a basis, $\beta'$. Prove that you found a basis.

2. Using the basis $\beta'$ you found in problem A, write the vectors $(1, 0, 0, 0)$ and $(1, 1, 1, 1)$ in $\beta'$-coordinates. (Find $[(1, 0, 0, 0)]_{\beta'}$ in the book’s notation.)

3. Generalizing your answer to 1b, find a basis for the vector space $V$ of all $n \times n$ upper triangular matrices. What is $\dim V$?

4. For each of the matrices $A$ below determine $\dim(\text{Nul} A)$, $\dim(\text{Col} A)$, and $\dim(\text{Nul} A) + \dim(\text{Col} A)$

$$\begin{bmatrix} 1 & 2 & 6 & 1 \\ -2 & 0 & 4 & 2 \\ 4 & 1 & 0 & -2 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 3 & 3 & 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 4 & 6 & 8 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & -3 & -4 \end{bmatrix}$$

5. If $\beta$ spans a subspace $H$ of $\mathbb{R}^3$ and $\alpha$ spans a subspace $K$, then $\beta \cup \alpha$ spans $H + K$. Using this, find subspaces $H$ and $K$ of $\mathbb{R}^3$ such that $\dim(H + K) \neq \dim H + \dim K$. 