1. Let
\[ \alpha = [e_1, e_2] \quad \beta = [(1, 1), (0, 2)] \quad \gamma = [(0, 1), (1, 0)] \]
a. Show these three sets are bases for \( \mathbb{R}^2 \).
b. Write down the change of coordinates matrices
\[ P_{\beta \leftarrow \alpha} \quad P_{\alpha \leftarrow \beta} \quad P_{\gamma \leftarrow \beta} \quad P_{\alpha \leftarrow \gamma} \]

2. Recall that the standard ordered basis \( \alpha \) for \( \mathbb{R}^n \) consists of the \( n \) vectors \( e_i = (0, \ldots, 1, \ldots, 0) \) where 1 is in the \( i \)th spot.
a. Show that the ordered set
\[ \beta = [e_2, e_3, e_1] \]
is a basis of \( \mathbb{R}^3 \). Find the change of coordinates matrix that sends \( \alpha \)-coordinates to \( \beta \)-coordinates.
b. Let
\[ \gamma = [e_3, e_1, e_2] \]
Find the change of coordinates matrix that sends \( \beta \)-coordinates to \( \gamma \)-coordinates.

3. Consider the ordered basis
\[ \beta = [(1, 2), (0, 1)] \]
a. Write the vector \((1, 0)\) in \( \beta \)-coordinates.
b. Write the vector \((3, 3)\) in \( \beta \)-coordinates.
c. Consider the linear transformation
\[ T(b_1) = b_1 - b_2 \quad \text{and,} \quad T(b_2) = -b_1 \]
Take a guess at what the matrix that describes this linear transformation is. Does your matrix satisfy,
\[ A \cdot b_1 = b_1 - b_2 \quad \text{and,} \quad A \cdot b_2 = -b_2? \]
Probably not. The problem is that you wrote down the matrix with respect to the standard basis. The matrix doesn’t work because it is assuming the inputs are in a different coordinate system.

With that in mind, try and figure out how to write a matrix \( [A]_\beta \) that does work.