1. Find the eigenvalues (simplify!) for the following matrices:

\[
\begin{bmatrix}
2 & 3 \\
1 & 2
\end{bmatrix}
\]
\[
\begin{bmatrix}
-3 & 3 \\
-7 & 4
\end{bmatrix}
\]
\[
\begin{bmatrix}
2 & 4 & 2 \\
-1 & 0 & 0 \\
0 & -7 & -3
\end{bmatrix}
\]

2. Let

\[
A = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\]
where \(a, b, c, d\) are arbitrary real numbers.

a. Find the characteristic polynomial of \(A\), \(p(\lambda)\).

b. Suppose the eigenvalues of \(A\) are \(\lambda_1\) and \(\lambda_2\). Then the characteristic polynomial of \(A\) factors as

\[
p(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)
\]

Prove, using what you did in part a, that

\[
\lambda_1 + \lambda_2 = a + d \\
\lambda_1 \lambda_2 = ad - bc
\]

3. a. Find the eigenvalues of the \(n \times n\) diagonal matrix

\[
\begin{bmatrix}
a_1 \\
\vdots \\
a_n
\end{bmatrix}
\]
where the \(a_1, \ldots, a_n\) are arbitrary real numbers, and everywhere else is a 0.

b. Find eigenvectors corresponding to the eigenvalues you found in part a.