Conceptual questions:

1. a. The characteristic polynomial of a matrix $A$ is

$$\det(A - \lambda I) = \lambda^4 - 2\lambda^3 + \lambda^2 - 2\lambda$$

Show that $A$ is diagonalizable.

b. Show that $A$ is not invertible, again only using the characteristic polynomial.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3x - 4y \\ 2x - 3y \\ 2x - 4y + z \end{pmatrix}$$

Find a basis $B$ of $\mathbb{R}^3$ such that the matrix of $T$ with respect to $B$ is

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(Hint: Write the matrix of $T$ in standard form. You are essentially diagonalizing this matrix. The matrix $M$ tells you the eigenvalues of $T$, so you can skip the first step.)

3. The eigenvectors and eigenvalues of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ are

$$b_1 = (3, 1, 2) \quad \text{with eigenvalue } 2$$
$$b_2 = (3, 1, 1) \quad \text{with eigenvalue } 0$$
$$b_3 = (0, 1, 0) \quad \text{with eigenvalue } -4$$

Pick a basis $B$ of $\mathbb{R}^3$ (any basis you want) and find the matrix for $T$ relative to $B$.

Computational questions:

4. Diagonalize

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$
5. Multiplying by $A$ defines a linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$. Let $B$ be the basis formed by the columns of $A$, and $C$ be the basis

$$C = [(1, 1, 0), (0, -1, 1), (0, 0, 1)]$$

Find the matrix for $T$ relative to the bases $B$ and $C$. 
