1. Let $A$ be an $n \times n$ matrix.
   
   a. If $A$ is diagonalizable, then $A^3$ is diagonalizable. Why?
   
   b. If $A$ is invertible, then $A^3$ is invertible. Why?

2. a. Find all $h$ such that the following matrix not diagonalizable

   $\begin{pmatrix} 1 & h \\ 1 & -1 \end{pmatrix}$

   b. Find all $h$ such that the following matrix not diagonalizable

   $\begin{pmatrix} 1 & h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

3. a. Define a linear transformation

   $T : \mathbb{P}_3 \rightarrow \mathbb{P}_2$

   $T : p(t) \mapsto p'(t)$

   For example, if $p(t) = t^3 - t$, then

   $T(p(t)) = p'(t) = 3t^2 - 1$

   Find the matrix for $T$ with respect to the bases $\mathcal{B} = \{1, t, t^2, t^3\}$ of $\mathbb{P}_3$ and $\mathcal{C} = \{1, t, t^2\}$ of $\mathbb{P}_2$.

   b. Multiplication by the matrix

   $A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$

   defines a linear transformation $\mathbb{R}^2 \rightarrow \mathbb{R}^2$. Find the matrix for this linear transformation with respect to the basis $\mathcal{B} = \{(-1, 1), (1, 1)\}$.

4. a. The transformation $x \mapsto Ax$ where

   $A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

   is the composition of a scaling and rotation. Find the angle of rotation $\varphi$ and the scaling factor $r$.

   b. Find the eigenspaces of the matrix

   $B = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

   and prove they are orthogonal (Hint: just check the basis vectors).