Math 111: Introduction to Probability
Quiz 10 — April 22nd, 2008 Name: ______________________________

This quiz covers material from **appendix D <u>and</u> section 8.5**. Show your work.

1. (*3 points*) A basketball stadium has enough seats for 20,000 fans. There are 100 seats that are broken in one way or another.

a. (*1 pt*) If you buy four tickets for you and your friends, what is the probability that everyone (exactly four people) has a broken seat?

Answer: Of the 20,000 seats, four are chosen. There are 100 bad seats and therefore 19,900 good seats. So,

$$P(X = 4) = \frac{C(100, 4) \cdot C(19900, 0)}{C(20000, 4)}$$

b. (*1 pt*) If you buy four tickets for you and your friends, what is the probability that someone (at least one person) has a broken seat?

Answer: The converse event to "at least one" is "none". So,

$$P(X \geq 0) = 1 - P(X = 0) = 1 - \frac{C(100, 0) \cdot C(19900, 4)}{C(20000, 4)}$$

c. (*1 pt*) Section F contains 300 seats. What is the expected number of broken seats in section F?

Answer: Think of this as picking 300 seats. Then for any hypergeometric random variable,

$$E(X) = \frac{(\text{\# of objects picked}) \cdot (\text{\# of bad objects})}{(\text{total \# of objects})} = \frac{300 \cdot 100}{20000} = 1.5$$

2. (*3 points*) Suppose X is a normal random variable with $\mu = 20$ and $\sigma = 12$. Find the value of

a. (*2 pts*) $P(X < 35)$

Answer:

$$P(X < 35) = P\left(Z < \frac{35 - 20}{12}\right) = P(Z < 1.25)$$

Looking up the value for 1.25 on the table, we get that the answer is .8944.

b. (*1 pt*) $P(X > 8)$

Answer: Done the same way, but at the last step we need to use the fact that for a standard normal variable Z, $P(Z > t) = P(Z < -t)$:

$$P(X > 8) = P\left(Z < \frac{8 - 20}{12}\right) = P(Z > -1) = P(Z < 1) = .8413.$$

3. (*2 points*) Suppose Z is a standard normal variable. Find the value of z if z satisfies $P(Z > z) = .2611$.

Answer: From the table, we get that $P(Z < -.64) = .2611$. Thus $P(Z > .64) = .2611$, so the required value of z is

$$z = .64$$