

HW1 Problem B for Math 420

You are a consultant for a company whose factory has been releasing a pollutant chemical into a lake at a constant rate for a long time. The factory is the sole source of this pollutant in the lake, and is located near a river that provides most of the water flow into the lake. Water flows out of the other end of the lake at a rate of 7.4 cubic meters per second. The lake has a volume of 110,000,000 cubic meters.

Government inspectors measure the pollutant concentration at the outflow of the lake to be 0.82 ppm (parts-per-million), and inform the company that owns the factory that this concentration must be reduced to at most 0.3 ppm by the time of a follow-up inspection one year later. The company decides that it is economically infeasible to defy the government or to modify its factory to produce less of the pollutant per unit time, and instead decides to shut down the factory for part of the year in order to pass the inspection. Your job is to recommend to the company what time period(s) to shut down the factory in order to minimize the time the factory is idle while still passing the inspection.

1. Construct a preliminary model that expresses how the pollutant concentration varies over time, both during time periods when the factory is operating and during periods when it isn't, without requiring any additional data. Discuss the assumptions behind this model.
2. According to your preliminary model, if the factory continues operating for the first part of the year and then shuts down for the rest of the year, how long can it operate without failing the year-end inspection?
3. What if the factory shuts down immediately and then resumes operation later in the year – how early can it resume operation and still pass the inspection?
4. Is there another strategy that would allow the factory to operate for more days over the course of the year? What recommendation do you make to the company for the immediate future, while you work on refining your model?
5. When the company first contacted you, you asked them to repeat the inspector's measurement and to measure the rate of the factory's pollutant output into the lake. They now report to you that the inspector's measurement has been confirmed to two decimal places, and that the factory outflow is 640 cubic meters of water per day with a pollutant concentration of 970 ppm. Is this data consistent with your preliminary model? If not, modify your model to make it consistent, and adjust your recommendations accordingly.
6. One week after shutting down the factory, the measured pollutant concentration drops from 0.82 ppm to 0.75 ppm. Again, is this data consistent with your model, and if not how could you adjust your model and recommendations?
7. Discuss other data that would be useful in refining your model, and other factors that might affect your final recommendations.