# More Group Theory Problems

Items marked with an asterisk (*) are problems that will be covered if time permits.

The following problems are from Hungerford's book:

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|---|---|
| Page 29 | # 7, 10*, 14* |
| Page 33 | # 5*, 15, 18* |
| Page 36 | # 6*, 7 |
| Page 40 | # 4, 11* |
| Page 45 | # 16 |
| Page 51 | # 3, 12 |
| Page 59 | # 8 |
| Page 63 | # 6, 14 |
| Page 68 | # 3, 8* |
| Page 75 | # 9*, 11 |
| Page 81 | # 1, 9 |
| Page 91 | #9 |
| Page 96 | # 2, 7 |
| Page 107 | #4 |
| Page 111 | # 1, 3*, 9 |

Additionally, try the following:

1. Suppose $K \lhd G$. Consider $K \lhd G/H$.
   Prove that there exists a unique group $\widetilde{K} < G$ so that $\widetilde{K}/H = K$ and $\widetilde{K} \lhd G$.

2. Show (by example, not by appealing to uniqueness of free objects) that $i : \{*\} \to \mathbb{Z}/(n)$ where $i(*) = 1$ is not a free group on {*}.

3. Let G be a group. Show that $\rho : G \to Aut(G)$ is a homorphism

4. Let G and H be groups and let N be a normal subgroup of $G \times H$. Suppose that $N \cap (G \times 1) = 1$ and $N \cap (1 \times H) = 1$. Show that N is contained in the center of $G \times H$.

5. (a) Let $p$ and $q$ be distinct primes, and let $S_n$ be the symmetric group on $n$ letters. Show that $S_n$ contains an element of order $pq$ if and only if $n \geq p + q$.
   (b) Let $p$ be an odd prime, let $n = p + 2$ and let $A_n$ be the subgroup of $S_n$ consisting of even permutations. Does $A_n$ contain an element of order $2p$? Why or why not?