Linear Algebra Mini-Sheet Number 1

Math SPIRAL Tutorial

June 4, 2004

1. Prove that for a vector $v \in V$ and a scalar $\alpha \in \mathbb{F}$,
   (a) $Ov = 0$
   (b) $(-1)v = -v$

2. Let V be the set of all pairs (x, y) where $x, y \in \mathbb{R}$ and let $\alpha \in \mathbb{R}$. Define

$$(x, y) + (x_1, y_1) = (x + x_1, y + y_1)$$
$$\alpha(x, y) = (\alpha x, y)$$

   With these operations, is V a vector space over $\mathbb{R}$? If so, prove it. If not, find operations such that V is a vector space.

3. Let $\mathbb{F}$ be a field and $n \in \mathbb{Z}$ let $V = M_{n \times n}(\mathbb{F})$. Which of the following matrices are subspaces of V?
   (a) all invertible $A \in V$.
   (b) all non-invertible $A \in V$
   (c) all $A \in V$ such that $AB = BA$ for some fixed matrix $B \in V$
   (d) all $A \in V$ such that $A^2 = A$

4. Let U, W be subspaces of a vector space V. Prove that $U \cap W$ is also a subspace of V.

5. Are the vectors

$$v_1 = (1, 1, 2, 4) \qquad v_2 = (2, -1, -5, 2)$$
$$v_3 = (1, -1, -4, 0) \qquad v_4 = (2, 1, 1, 6)$$

   linearly independent in $\mathbb{R}^4$?