

Linear Algebra

Math SPIRAL Tutorial

June 10, 2004

1. Given the system of equations

$$\begin{aligned}x - 2y + 3z &= 0 \\2x - y - z &= 0 \\-3x - 4y + 5z &= 0\end{aligned}$$

What is $NS(A)$?

2. Consider the vectors

$$\mathbf{v}_1 = (1, -2, 3) \quad \mathbf{v}_2 = (-1, -4, 9) \quad \mathbf{v}_3 = (-1, 0, 1)$$

- (a) Describe the set $P = \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$
 - (b) Find a basis for P .
 - (c) What is $\dim P$?
3. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that

$$\begin{aligned}(1, 1) &\mapsto (3, 4) \quad \text{and} \\(-1, 1) &\mapsto (2, 2)\end{aligned}$$

Find: $T(3, 1)$

4. Let E and F be vector spaces, not necessarily finite dimensional. Let $W : E \rightarrow F$ be a linear transformation. Show that $W(E)$ is a subspace of F .
5. Let a_1, a_2, \dots, a_n be distinct real numbers. Show that the n exponential functions $e^{a_1 t}, \dots, e^{a_n t}$ are linearly independent over \mathbb{R} . Hint: Think of $e^{a_i t}$ as a linear transformation (what does problem 4 say?) and use induction.