

# RECENT ADVANCES ON HYPERBOLIC CONSERVATION LAWS

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ABSTRACT. I will review recent results on nonlinear hyperbolic systems of partial differential equations that arise from fundamental conservation principles. The objective will be to present several tools (kinetic relation, family of paths, graph solutions,  $(\Phi, \psi)$ -admissible solutions) to tackle hyperbolic problems for which the effect of small scales on the selection of physically meaningful, discontinuous solutions is essential.

For instance, we will be interested in characterizing zero diffusion-dispersion limits for hyperbolic systems that are strictly hyperbolic but not globally genuinely nonlinear, and for systems of mixed (hyperbolic-elliptic) type. Solutions typically contain “nonclassical” waves, e.g. undercompressive shocks or phase transitions. These waves are fundamental in phase transition dynamics (van der Waals fluids, martensitic materials) when both viscosity and capillarity effects come into play.

While “classical” shocks are compressive, independent of small-scale regularization mechanisms, and can be characterized by a single entropy inequality, by contrast nonclassical waves are undercompressive, are very sensitive to diffusive and dispersive mechanisms. The selection of the latter is more delicate and requires an additional jump condition, referred to as a kinetic relation.

The following topics will be discussed. The role of the kinetic relation for solving the Riemann problem. The derivation of kinetic relations from a traveling wave analysis. The existence of nonclassical entropy solutions via Glimm scheme and wave front tracking. The uniqueness of  $(\Phi, \psi)$ -admissible solutions in the class of solutions with tame variation. The geometric approach and the existence of graph solutions. The nucleation criterion for thin films. The design of difference schemes based on entropy conservative flux.