Practice midterm

1) Compute the Frenet apparatus (κ, τ, T, N, B) for the following curve

$$\alpha(t) = (\frac{1}{2}e^t(\sin t + \cos t), \frac{1}{2}e^t(\sin t - \cos t), e^t)$$

2) Consider the spherical coordinates parametrization of the unit sphere $x^2 + y^2 + z^2 = 1$.

$$\phi(u, v) = (\cos u \cos v, \cos u \sin v, \sin u)$$

Let $p = (1/2, 1/2, 1/\sqrt{2})$. Verify that $v = (1, -1, 0)$ is tangent to the sphere at p and find the coordinates of v with respect to the natural basis of T_pS coming from ϕ.

3) Let S be the part of the round cone given by the equation $x^2+y^2-z^2 = 0$ with $0 < z < 1$.
   (a) Verify that S is a regular surface.
   (b) Find the tangent plane to S at $p = (1/2, 1/2, 1/\sqrt{2})$.
   (c) Let F be the rotation by $\pi/4$ around the z-axis. Show that F is a diffeomorphism of S onto itself
   (d) Compute the differential dF_p with respect to the parametrization of S as a graph $z = \sqrt{x^2 + y^2}$.
   (e) Compute the first fundamental form of S with respect to this parametrization.
   (f) Compute the area of S using the above parametrization.

4) Prove that if all principal normals to a space curve pass through the same point then this curve lies on a sphere.