Solution of Problem 3 from Midterm 2

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Make the trigonometric substitution $2\sec u$, $dx = 2\sec u \tan u du$

$$\begin{aligned}
\int \frac{\sqrt{x^2-4}}{x} dx &= \int \frac{\sqrt{4\sec^2 u - 4}}{2\sec u} 2\sec u \tan u du \quad (5 \text{ points}) \\
&= \int \sqrt{4\tan^2 u} \tan u du = \int 2\tan^2 u du \quad (5 \text{ points}) \\
&= \int (2\sec^2 u) du = 2\int \sec^2 u du - 2\int du \quad (5 \text{ points}) \\
&= 2\tan u - 2u + C \quad (5 \text{ points}) \\
&= \sqrt{x^2-4} - 2\cos^{-1}\left(\frac{2}{x}\right) + C \quad (5 \text{ points})
\end{aligned}$$

The last equality comes from

$$\begin{aligned}
&2\tan u = \sqrt{(2\sec u^2 - 4)} = \sqrt{x^2-4}; \\
&x = 2\sec u = \frac{2}{\cos u} \iff \cos u = \frac{2}{x} \iff u = \cos^{-1}\left(\frac{2}{x}\right).
\end{aligned}$$