MATH 141 - Exam 1, Problem 5, Solution

1. (20 points) Find the value of the following limit, if it exists:

$$\lim_{x \to 0+} \frac{x^3}{x - \sin(x)}$$

If the limit does not exist, explain why.

Solution:
$$\frac{0}{0}$$
, we can apply L'Hospital's Rule. (5 points)

$$\lim_{x \to 0+} \frac{x^3}{x - \sin(x)} = \lim_{x \to 0+} \frac{3x^2}{1 - \cos(x)} \quad \text{still } \frac{0}{0} \quad (5 \text{ points})$$

$$= \lim_{x \to 0+} \frac{6x}{\sin(x)} \quad \text{still } \frac{0}{0} \quad (5 \text{ points})$$

$$= \lim_{x \to 0+} \frac{6}{\cos(x)} = 6 \quad (5 \text{ points})$$