

Test Minimization Program

Instead of minimizing the energy function $E(y) = \int_{\Omega} \Phi(\nabla y(x)) dx$
with energy density

$$\Phi(C) = \kappa_1 (\text{tr}C - 2)^2 + \kappa_2 C_{12}^2 + \kappa_3 \left(\left(\frac{C_{11} - C_{12}}{2} \right)^2 - \varepsilon^2 \right)^2$$

I tested my minimization program with another function

$$f(\bar{u}) = \sum_{i=1}^{N-1} [\arctan(u_i) \sin(u_i) + \arctan(u_{i+1}) \cos(u_{i+1})]$$

- This is a multi-dimensional function.
- This function has many local minimizers.
- I can choose any arbitrary dimension.
- I can simply use `fminsearch` in Matlab to compute the local minimizer before waiting too long.

Compare my result with the one computed by Matlab

- Matlab

output minimizer is

4.7435
3.9719
3.9719
3.9719
3.9721
3.9719
3.9722
3.9720
3.9719
3.2111

output min function value
is

-17.5942

iteration number is

436

- My Conjugate Gradient Method

output minimizer is

4.74362
3.97198
3.97198
3.97198
3.97198
3.97198
3.97198
3.97198
3.97198
3.2111

output minimum function value is

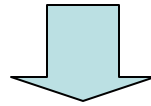
-17.5942

iteration number is

36

Test simulation program

$$\Phi(C) = \kappa_1 (\text{tr}C - 2)^2 + \kappa_2 C_{12}^2 + \kappa_3 \left(\left(\frac{C_{11} - C_{12}}{2} \right)^2 - \varepsilon^2 \right)^2$$



$$\Phi(C) = \kappa_1 (C_{11} - 1)^2 + \kappa_2 (C_{12}^2 - \varepsilon^2)^2 + \kappa_3 (C_{22} - 1 - \varepsilon^2)^2$$

This density function satisfies the properties of energy density:

- Frame indifference: $\Phi(RF) = \Phi(F) \quad \forall R \in SO(3)$

- $\Phi(F) \geq 0$

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$$\Phi(C) = 0 \Leftrightarrow \begin{cases} C_{12} = C_{21} = \pm \varepsilon \\ C_{11} = 1 \\ C_{33} = 1 + \varepsilon^2 \end{cases}$$

Benefits

- This energy density has two minimizing deformation gradients,

$$U_0 = \begin{pmatrix} 1 & \varepsilon \\ 0 & 1 \end{pmatrix}, U_1 = \begin{pmatrix} 1 & -\varepsilon \\ 0 & 1 \end{pmatrix}$$

$$U_0 - U_1 = \begin{pmatrix} 0 & 2\varepsilon \\ 0 & 0 \end{pmatrix} = 2\varepsilon \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 2\varepsilon \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes e_2$$

Thus, the laminate will be parallel to x-axis.

Local minimizer or Global minimizer

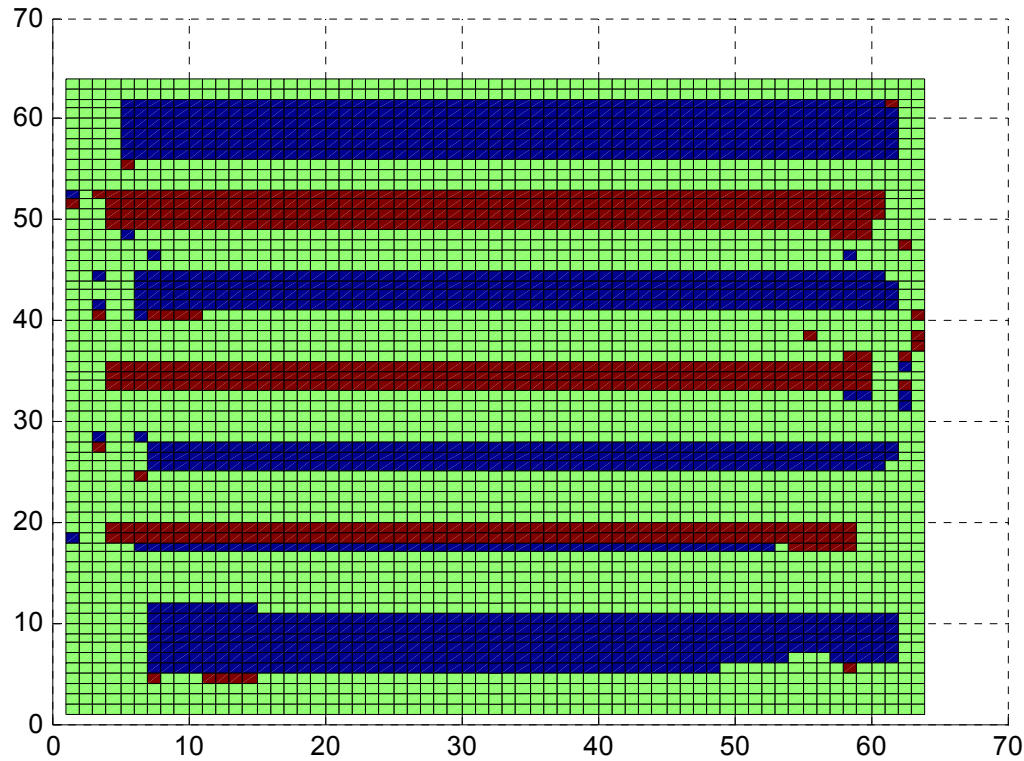
- Numerical method is too expensive
- In the test program, we can set initial guess to be the small oscillations of the theoretical minimizers of energy density function.

Bugs in boundary condition

$$y(x) = F_{\lambda} x \quad \text{For } x \in \partial\Omega$$

- Fix deformation of each node on the boundary.
- Fix deformation gradient of each element on the boundary. (Wrong)

Result for 64*64 mesh



This is not a proper result coming from the global minimizer. But we can still see the right pattern from the local minimizer.