1. (20pts) Find the solution of the following IBVP:

\[ u_{tt} - 4u_{xx} = \cos(t) \quad 0 < x < \infty, \ t > 0 \]
\[ u(x, 0) = \sin(x), \quad u_t(x, 0) = 0, \]
\[ u(0, t) = \sin(t). \]

2. (30pts) Using the reflection method (with an even reflection), find a formula for the solution of the Neumann problem for the wave equation on the half-line:

\[ u_{tt} - c^2u_{xx} = 0 \quad 0 < x < \infty, \ t > 0 \]
\[ u(x, 0) = \phi(x), \quad u_t(x, 0) = \psi(x), \]
\[ u_x(0, t) = 0. \]

3. (30pts) Consider the diffusion equation \( u_t - ku_{xx} = 0 \) on \((0, L)\) with Robin boundary conditions \( u_x(0, t) - a_0 u(0, t) = 0 \) and \( u_x(L, t) + a_L u(L, t) = 0 \). If \( a_0 > 0 \) and \( a_L > 0 \), use the energy method to show that the endpoints contribute to the decrease of the energy \( \int_0^L u(x, t)^2 \, dx \).

4. (20pts) Let \( u \) and \( v \) be such that

\[ u_t - u_{xx} = f \quad \text{for} \ 0 < x < L, \ t > 0 \]

and

\[ v_t - v_{xx} = g \quad \text{for} \ 0 < x < L, \ t > 0 \]

with \( f(x, t) \leq g(x, t) \) for all \( x, t \), and with \( u \leq v \) at \( x = 0, x = L \) and \( t = 0 \). Prove that \( u(x, t) \leq v(x, t) \) for \( x \in [0, L], \ t > 0 \).