1. (30 pts) Solve the following BVP in the rectangle \( \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\} \):
\[
\begin{cases}
\Delta u = 0 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\
u_x(0, y) = 0 & 0 \leq y \leq 1, \quad u_x(1, y) = \cos(\pi y) + \cos(2\pi y) & 0 \leq y \leq 1 \\
u_y(x, 0) = 0 & 0 \leq x \leq 1, \quad u_y(x, 1) = 0 & 0 \leq x \leq 1
\end{cases}
\]

2. (30 pts) Find the solution of the following BVP in the semi-infinite strip \( \{(x, y) : 0 \leq x \leq 1, y \geq 0\} \)
\[
\begin{cases}
u_x(0, y) = 0, & u_x(1, y) = 0 \quad \text{for all } y \geq 0 \\
u(x, 0) = x & \text{for all } 0 \leq x \leq 1,
\end{cases}
\]
with \( u(x, y) \) bounded (that is \( |u(x, y)| \leq M \) for some \( M \)).

3. (40 pts) Find the solution of Laplace’s equation in the semi-infinite strip \( \{(x, y) : 0 \leq x \leq 2, y \geq 0\} \) satisfying the following mixed boundary conditions:
\[
\begin{cases}
u(0, y) = 0, & u_x(2, y) = 0 \quad \text{for all } y \geq 0 \\
u(x, 0) = 2 \sin(3\pi x/4) - 3 \sin(7\pi x/4) & \text{for all } 0 \leq x \leq 2, \\
\lim_{y \to +\infty} u(x, y) = 0
\end{cases}
\]