1. (25pt) Consider waves in a resistant medium that satisfy the problem

\[ u_{tt} - c^2 u_{xx} = -ru_t \quad 0 < x < L, \quad t > 0 \]

\[ u(0, t) = 0, \quad u(L, t) = 0 \]

where \( r \) is a constant \( 0 < r < 2\pi c/L \). Use the separation of variables method to find a series solution of this boundary value problem.

2. (a) (25pt) Find the values of \( \lambda \) for which the following boundary value problem has non-trivial solutions:

\[ X'' + \lambda X = 0 \quad \text{for} \quad 0 < x < 1, \quad X(0) = 0, \quad X'(1) = 0. \]

For each such \( \lambda \), find the corresponding solutions \( X(x) \).

(b) (25pt) Use the separation of variables method and your answer to the question above to find the solution of the following mixed boundary problem:

\[ u_t - ku_{xx} = 0 \quad 0 < x < 1, \quad t > 0 \]

\[ u(0, t) = 0, \quad u_x(1, t) = 0 \]

\[ u(x, 0) = \sin\left(\frac{3\pi}{2} x\right) - 2 \sin\left(\frac{5\pi}{2} x\right) \]

3. (25pt) Find the eigenvalues graphically for the boundary problem

\[ X'' + \lambda X = 0 \quad 0 < x < L \]

\[ X(0) = 0, \quad X'(L) + X(L) = 0 \]