1. (20pt) For each of the following functions, state whether it is even or odd or periodic. If periodic, what is the smallest period?

   (a) $\tan(ax)$ ($a > 0$)
   (b) $\sin(x^2)$
   (c) $e^{-x}$
   (d) $x^m$ ($m =$integer)
   (e) $\frac{\sin(x)}{2 + \cos(x)}$

2. (30) Compute the full Fourier series of the function $f(x) = |x|$ on the interval $(-L, L)$. Sketch the function to which the Fourier series converges on the interval $(-3L, 3L)$.

3. (25pt)

   (a) Let $\phi(x)$ be a continuous function on $(0, L)$. Under what conditions is its odd extension also a continuous function?

   (b) Same as part (a) for the even extension.

4. (25pt) We saw in Homework #3 that the boundary conditions $X(0) = 0$, $X'(1) = 0$ lead to the eigenfunctions

   $$X_n(x) = \sin \left( \left( n + \frac{1}{2} \right) \pi x \right) \quad n = 0, 1, 2, \ldots$$

   Assuming that a function $\phi(x)$ defined on $(0, 1)$ can be written as

   $$\phi(x) = \sum_{n=0}^{\infty} c_n \sin \left( \left( n + \frac{1}{2} \right) \pi x \right)$$

   find a formula for the coefficient $c_n$ as an integral of $\phi$. 