1. (30pts) Let $\gamma_n$ be a sequence of constants tending to $\infty$. Let $f_n(x)$ be a sequence of functions defined as follows:

\[ f_n(1/2) = 0 \]
\[ f_n(x) = \gamma_n \text{ in the interval } \left[ \frac{1}{2} - \frac{1}{n}, \frac{1}{2} \right) \]
\[ f_n(x) = -\gamma_n \text{ in the interval } \left( \frac{1}{2}, \frac{1}{2} + \frac{1}{n} \right] \]
\[ f_n(x) = 0 \text{ elsewhere.} \]

Show that:

(a) $f_n(x)$ converges to 0 pointwise.
(b) The convergence is not uniform.
(c) $f_n(x)$ converges to 0 in the $L^2$ sense if $\gamma_n = n^{1/3}$.
(d) $f(x)$ does not converge in the $L^2$ sense if $\gamma_n = n$.

2. (30pts) Let

\[ \phi(x) = \begin{cases} 
-1 - x & \text{for } -1 < x < 0 \\
1 - x & \text{for } 0 < x < 1 
\end{cases} \]

(a) Find the full Fourier series of $\phi(x)$ in the interval $(-1, 1)$.
(b) Does it converge in the mean square sense?
(c) Does it converge pointwise?
(d) Does it converge uniformly to $\phi(x)$ in the interval $(-1, 1)$?

3. (20pts) Let $f(x) = \cosh(x)$, $-\pi \leq x \leq \pi$.

(a) Find the full Fourier series of $f$ (recall that $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$).
(b) For which values of $x$ does the Fourier series of $f$ converge to $f$?
(c) Use (b) to evaluate the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$.

4. (20pts) Find the solution of the following IBVP:

\[ u_{tt} - u_{xx} = 0 \quad \text{for } 0 < x < 1, \quad t > 0 \]
\[ u(0, t) = 0, \quad u(1, t) = 0 \]
\[ u(x, 0) = x, \quad u_t(x, 0) = 1 \]