1. (30 pt) Let $u(x,t)$ be the solution of

$$
\begin{align*}
    u_{tt} - 4u_{xx} &= 0 & 0 < x < \infty, \ t > 0 \\
    u(x,0) &= \phi(x), & u_t(x,0) = \psi(x), \\
    u(0,t) &= 0
\end{align*}
$$

with $\phi$ given by

and $\psi = \begin{cases} 
    1 & \text{if } 1 < x < 3 \\
    0 & \text{otherwise}
\end{cases}$.

Using the domain of influence, determine

(a) The time at which $u(10,t)$ becomes non-zero for the first time.

(b) The time after which you are sure that $u(10,t)$ will always be zero.

You do not have to solve the PDE.

2. (40 pt) Let $f(x,t)$ be any function and let

$$
    u(x,t) = \frac{1}{2c} \iint_{\Delta} f = \frac{1}{2c} \int_0^t \int_{x+ct cs}^{x-ct+cs} f(y,s) \, dy \, ds
$$

where $\Delta$ is the usual triangle of dependence of $(x,t)$. Verify directly by differentiation that

$$
    u_{tt} - c^2 u_{xx} = f, \quad u(x,0) = 0, \quad u_t(x,0) = 0.
$$

3. (30 pt) Find the solution of

$$
\begin{align*}
    u_{tt} - c^2 u_{xx} &= e^x & -\infty < x < \infty, \ t > 0, \\
    u(x,0) &= 0, \\
    u_t(x,0) &= \cos(x).
\end{align*}
$$