Calculus 221, section 9.3 Definite Integrals
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Back in Math 220, section 6.3, we made use of the Fundamental Theorem of Calculus:
Given a function \( f(x) \) which is continuous on an interval \([a, b]\), and given \( F(x) \) which is an antiderivative of \( f(x) \),
then

\[
\int_{a}^{b} f(x) \, dx = F(b) - F(a) = F\left( b \right) - F \left( a \right).
\]

Note that this theorem can make use of any antiderivative, so we’ll generally choose the easiest version, i.e. the one without the “+ C”. This works because we’d always get “+ C – C = 0” anyway.

Example A: Evaluate \( \int_{0}^{\pi/6} \cos(3t) \, dt \). \( \text{answer: } -\frac{1}{3} \)

Example B: Find \( \int_{1}^{2} \frac{\ln x}{2x} \, dx \). \( \text{answer: } 1 \)

Example B using the Change of Limits Rule: Find \( \int_{1}^{2} \frac{\ln x}{2x} \, dx \). \( \text{answer: } 1 \)

Example C: \( \int_{0}^{2} \frac{x}{(x^2 + 3)^5} \, dx \). \( \text{answer: } -\frac{1}{8} \left[ \frac{1}{7^4} - \frac{1}{3^4} \right] \approx 0.0014911482 \)
Example D: \[ \int_{1}^{1.5} 5x e^{x^2} - 1 \, dx \].  \text{answer:} \frac{5}{2} \left[e^{1.25} - 1\right]

Example E: \[ \int_{0}^{10} \frac{x^2}{x^3 + 8} \, dx \].  \text{answer:} \frac{1}{3} \left[\ln 1008 - \ln 8\right] = \frac{1}{3} \ln(126)

Example F: \[ \int_{1}^{2} x^3 \ln x \, dx \].  \text{answer:} 4 \ln 2 - \frac{15}{16}

Example G: Find the area under the curve \( y = x^2 \cos x \) on the interval \(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\).  \text{answer:} \frac{\pi^2}{2} - 4