A review of what we have so far: For a geometric series, whenever $|r| \geq 1$, the series is divergent. But whenever $|r| < 1$, the series is convergent and
\[ a + ar + ar^2 + ar^3 + ar^4 + \ldots = \frac{a}{1 - r}. \]

Example J: A perpetuity will pay $10,000 annually, with a rate of interest compounded annually. How much must be deposited now to cover these payments? (That is, what is the present value of this perpetuity?)

*answer*: $135,000

Example K: In 2001, to stimulate the U.S. economy, the IRS issued “advance payments of a 2001 tax credit”, amounting to about $38 billion sent to taxpayers. One study found that, over a 6 month period, households spent 63% of their windfall on goods and services. (source: [wws.princeton.edu/econdp](http://wws.princeton.edu/econdp)) The recipients of this money then spent 63% of what they received. Assume this pattern is repeated again and again, and 63% of the money received is spent at each stage. How much total spending could be attributed to the original $38 billion in payments?

*answer*: $64,702,702,702
Example L: Quinine was developed around 1630 by Jesuits in Peru as a treatment for malaria, and this drug is still the primary treatment today. Suppose a person is given a 50 mg dose of quinine at the same time every day as a prophylactic (i.e. preventive treatment), and one’s body metabolizes 77% of the quinine in one’s body each day. In the log run, the amount of quinine present stabilizes at what amount?

*answers:* 65 mg right after a dose and 15 mg just before a dose.

Other applications include tracking the accumulations of toxins in the body and depletion of natural resources. Some of you may already be familiar with “sigma notation” to indicate a sum. It is also appropriate for series.

\[ \sum_{k=0}^{10} a_k = a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} \]

\[ \sum_{k=0}^{\infty} a_k = a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + \ldots \]

Example M: Does the infinite series \( \sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \) converge, and if so, to what value?

*answer:* 2

Example N: Evaluate \( \sum_{k=0}^{\infty} \frac{1}{3} \left( \frac{2}{3} \right)^k \). *answer:* 1
Example O: Evaluate \( \sum_{k=1}^{\infty} \frac{4}{3^k} \). \textit{answer: 2}

Example P: Evaluate \( \sum_{k=0}^{\infty} \frac{(-1)^k}{3^{2k}} \). \textit{answer: } \frac{9}{10}

Note that we can take the result (stated earlier) for a geometric series and write it using sigma notation.

Whenever \( |r| < 1 \), \( \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} \).

We can also express an \( n \)th Taylor polynomial in sigma notation as a partial sum:

\[
p_n(x) = \frac{f(0)}{0!} + \frac{f^{(1)}(0)}{1!} x + \frac{f^{(2)}(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 + \ldots + \frac{f^{(n)}(0)}{n!} x^n = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k.
\]